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Distributions of polynomials in Gaussian random variables: recent progress and open problems

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This lecture is based on our joint papers [1] and [2] with E.D. Kosov and G.I. Zelenov. A recent survey of results about distributions of polynomials on finite- and infinite-dimensional spaces with measures is given in [3].

We recall that the Nikolskii-Besov class $B^\alpha(\mathbb{R}^k)$ of order $\alpha \in (0,1)$ consists of all functions $\varphi \in L^1(\mathbb{R}^k)$ such that
\[
\|\varphi(\cdot + h) - \varphi\|_{L^1} \leq C|h|^\alpha \quad \forall h \in \mathbb{R}^k
\]
for some number $C$. The minimal possible $C$ will be denoted by $\|\varphi\|_{B^\alpha}$. In case of a measure $\mu$ with a density $\varphi$ of class $B^\alpha(\mathbb{R}^k)$ we set $\|\mu\|_{B^\alpha} = \|\varphi\|_{B^\alpha}$. Let $\| \cdot \|_{TV}$ denote the total variation norm of a measure and let $d_K$ be the Kantorovich distance on the set of probability measures.

**Theorem.** Let $\nu$ and $\sigma$ be two Borel probability measures on $\mathbb{R}^k$ with densities of class $B^\alpha(\mathbb{R}^k)$. Then
\[
\|\sigma - \nu\|_{TV} \leq C(k, \alpha)\|\sigma - \nu\|_{B^\alpha}^{1/(1+\alpha)} d_K(\sigma, \nu)^{\alpha/(1+\alpha)},
\]
where $C(k, \alpha)$ is a number depending on $k$ and $\alpha$.

We shall briefly discuss some recent progress in the study of Nikolskii-Besov classes following joint papers in preparation with E.D. Kosov and S.N. Popova.

The next result shows that the joint distribution density of several polynomials in Gaussian random variables, whenever it exists, always belongs to a fractional Nikolskii-Besov class that does not depend on the number of variables, but only on the number of polynomials and their maximal degree.

**Theorem.** Let $f = (f_1, \ldots, f_k) : \mathbb{R}^n \to \mathbb{R}^k$ be a mapping whose components $f_i$ are polynomials of degree $d$. Let $\gamma_n$ be the standard Gaussian measure on $\mathbb{R}^n$. Suppose that its image $\gamma_n \circ f^{-1}$ under $f$ has a density. Then this density belongs to $B^\alpha(\mathbb{R}^k)$ for every $\alpha < (4k(d-1))^{-1}$.

If $k = 1$ and $f$ is not a constant, then the density of $\gamma_n \circ f^{-1}$ belongs to $B^{1/d}(\mathbb{R})$.

Infinite-dimensional analogs and challenging open problems will be also discussed.
Acknowledgement: The work was supported by the RSF Grant 17-11-01058.

References


Non-backtracking spectrum of random graphs

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In this talk, we will introduce the Hashimoto’s non-backtracking matrix. We will illustrate on sparse random matrix ensembles how this non-Hermitian matrix can be used as a powerful tool to compute spectral radii and spectral gaps.

On the small noise limit for some nonlinear SPDEs with vanishing noise correlation

Sandra CERRAI University of Maryland, USA, E-mail: cerrai@gmail.com

We are dealing with the study of the validity of a large deviation principle for some nonlinear PDEs, perturbed by a Gaussian random forcing. We are here interested in the regime where both the strength of the noise and its correlation are vanishing, on a length scale $\epsilon$ and $\delta$, respectively, with $0 < \epsilon, \delta << 1$. Depending on the relationship between $\epsilon$ and $\delta$ we will prove the validity of the large deviation principle in different functional spaces. We will illustrate our method by considering the two-dimensional stochastic Navier-Stokes equation and a class of stochastic reaction-diffusion equations, defined in any space dimension, including the dynamical $\Phi^4_{2n}$ model.

References

2D Ising model at criticality: correlations, interfaces, a priori estimates

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In this talk we survey recent results (joint with Hongler and Izyurov) on convergence of correlation functions in the critical 2D Ising model to the Conformal Field Theory predictions \cite{[1,2]} (see also the invited session talk by Izyurov); discuss convergence of domain walls to Schramm’s SLE curves \cite{[3]} and the full Conformal Loop Ensemble (the latter is a recent result due to Benoist and Hongler \cite{[4]}); and highlight the role of discrete analysis techniques, notably uniform Alhfors-Beurling style estimates \cite{[5]}, needed to prove uniform crossing estimates in rough domains \cite{[6]} and to control the discrete exploration procedure in \cite{[4]} when the mesh size goes to zero.

References


Geometry of random planar maps and stable processes

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We will survey recent results on the geometry of large random planar maps according to the weight given to their vertex degrees. The usual Brownian and stable paradigms occur in this setup with an additional phase transition within the stable regime. Although the Brownian case is by now fairly well understood - thanks to recent works on the Brownian map- the stable case remains much more elusive...and fascinating!
Limit shapes beyond dimers

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This talk is based on joint work with Jan de Gier (Melbourne) and Sam Watson (Brown University).

We discuss the 5-vertex model, a generalization of the lozenge tiling model in which one pair of lozenges interact. Equivalently, it is a special case of the six-vertex model where one vertex configuration is disallowed.

We compute the explicit free energy, surface tension function, and the Euler-Lagrange equation describing the limit shapes for the height function in the model. Since the model is not determinantal we use the Bethe Ansatz technique, which in this case has the convenient property that the Bethe roots have an explicit form.

Potential theory for symmetric jump processes and applications

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There has been a long history of research on heat kernel estimates and Harnack inequalities for diffusion processes. Harnack inequalities and Hölder regularities for harmonic/caloric functions are key components of the celebrated De Giorgi-Nash-Moser theory in harmonic analysis and partial differential equations. Yet, such a theory has been developed only recently for jump processes despite of the fundamental importance in analysis.

In this talk, I will summarize developments of the De Giorgi-Nash-Moser theory for symmetric jump processes and discuss its applications.

Acknowledgement: This talk is partly based on joint works with my collaborators; M.T. Barlow, R.F. Bass, Z.-Q. Chen, A. Grigor’yan, J. Hu, P. Kim, M. Kassmann and J. Wang.

An individual-based model for the Lenski experiment, and the deceleration of the relative fitness

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Abstract: The Lenski experiment investigates the long-term evolution of bacterial populations. Its design allows the direct comparison of the reproductive fitness of an evolved strain with its founder ancestor. It was observed by Wiser et al. (2013) that the mean fitness over time increases sublinearly, a behaviour which is commonly attributed to effects like clonal interference or epistasis. In this paper we present an individual-based probabilistic model that captures essential features of the design of the Lenski experiment. We assume that each beneficial mutation increases the individual reproduction rate by a fixed amount, which corresponds a priori to the absence of epistasis. Using an approximation by near-critical Galton-Watson processes, we prove that under some assumptions on the model parameters which exclude clonal interference, the relative fitness process converges, after suitable rescaling, in the large population limit to a power law function.
References


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**Energy Saving Approximation for Random Processes**

Mikhail LIFSHITS *St. Petersburg State University, Russia*, E-mail: mikhail@lifshits.org

We consider a stationary process or a process with stationary increments (with either discrete or continuous time) as a target and find an approximating process from the same class combining good approximation properties and, appropriately understood, small expense of energy. Our aim is to solve the problem in terms of spectral characteristics of approximated process. If there is no extra adaptivity assumptions on the approximating process, the problem is easy and admits a closed universal solution, which is however non-obvious even for approximation of i.i.d. sequences. Under adaptivity assumption, the problem has very much in common with classical prediction problems and solution construction depends on the spectrum of the approximated process. In this direction, we also extend classical spectral criteria for regularity and singularity of second order stationary processes due to Kolmogorov and Krein. This is a joint work with I.A.Ibragimov, Z.Kabluchko, and E.Setterqvist.

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**Point processes related to q-hypergeometric polynomials**

Grigori OLSHANSKI *Institute for Information Transmission Problems & Higher School of Economics & Skoltech, Russia*, E-mail: olsh2007@gmail.com

Orthogonal polynomials play an essential role in a number of probabilistic models arising in random matrix theory and representation theory. I will report on recent results concerning random point processes with long-range interaction between particles. The construction of these processes uses orthogonal polynomials living on a q-lattice, which leads to new effects. Partly based on joint work with Vadim Gorin.

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**SPDEs: Probability laws and trajectories**

Marta SANZ-SOLÉ *University of Barcelona, Spain*, E-mail: marta.sanz@ub.edu

**Key words:** Stochastic partial differential equations, Malliavin calculus, hitting probabilities

**Mathematical Subject Classification:** 60H07, 60G60, 60H15, 60J45, 60G15

A basic question in probabilistic potential theory is to determine whether a random field ever visits a fixed deterministic set $A$. This leads to a quantitative analysis of the hitting probabilities in terms of geometric measure.
notions, like the Hausdorff measure or the Bessel-Riesz capacity of the set $A$. For large classes of Lévy and Markov processes, there has been a lot of work since the 40’s, bringing the subject to a state of maturity. Initiated in [2], and motivated by [1], the study of hitting probabilities relative to sample paths of systems of SPDEs has been in the focus of interest and developed during the last years.

In the lecture, a method to approach this problem will be described. The role played by the dimension of the random field and the index set, the regularity of the sample paths, and the properties of the one-point and two-point joint densities, will be highlighted. We will discuss the importance of Malliavin calculus in the approach, in particular, for obtaining a qualitative behaviour of the two-point densities when these points get close to each other. We will illustrate the implementation of the method by surveying results on two classical examples: the nonlinear stochastic heat and wave equations. Finally, I will mention some on-going work on elliptic stochastic equations and open questions. The lecture is based on joint work with R.C. Dalang ([4], [6]) and many other contributions, among them, [3], [5], [7].

References


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A hierarchical renormalisation model

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I am going to discuss a few questions, with but more often without answers, about the free energy in a simplified model of depinning transition in the limit of strong disorder. The study, initiated by Derrida and Retaux in 2014, can also be formulated for an elementary percolation model on trees. Joint work with Xinxing Chen, Bernard Derrida, Yueyun Hu and Mikhail Lifshits.
Ergodicity of stochastic differential equations with jumps and singular coefficients

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Abstract: We show the strong well-posedness of SDEs driven by general multiplicative Lévy noises with Sobolev diffusion and jump coefficients and integrable drift. Moreover, we also study the strong Feller property, irreducibility as well as the exponential ergodicity of the corresponding semigroup when the coefficients are time-independent and singular dissipative. In particular, the large jump is allowed in the equation. To achieve our main results, we present a general approach for treating the SDEs with jumps and singular coefficients so that one just needs to focus on Krylov’s apriori estimates for SDEs. (This is a joint work with Longjie Xie.)
Stochastic invariance for SDEs and applications

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Bruno Bouchard  Université Paris-Dauphine, France,
Camille Illand  AXA Investment Managers, France.

We provide a new characterization of the stochastic invariance of a closed subset of $\mathbb{R}^d$ with respect to a diffusion. We extend the well-known inward pointing Stratonovich drift condition to the case where the diffusion matrix can fail to be differentiable: we only assume that the covariance matrix is. In particular, our result can be directly applied to construct affine and polynomial diffusions on any arbitrary closed set. In addition, we provide an extension to the jump-diffusion case and an equivalent formulation in the semimartingale framework.

References


Scaling limit of the recurrent random walk on a Galton-Watson tree

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Loïc de Raphélis  ENS Lyon, France

We consider a biased random walk on a Galton-Watson tree. This Markov chain is null recurrent for a critical value of the bias. In that case, Peres and Zeitouni [1] proved that the height of the Markov chain properly rescaled converges in law to a reflected Brownian motion. We show that the trace of this Markov chain converges in law to the Brownian forest.
Acknowledgement: ANR GRAAL, ANR LIOUVILLE

References


An HJB Approach to a General Continuous-Time Mean-Variance Stochastic Control Problem

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A general continuous mean-variance problem is considered where the cost functional has an integral and a terminal-time component. The problem is transformed into a superposition of a static and a dynamic optimization problems. The value function of the latter can be considered as the solution to a degenerate HJB equation either in viscosity or in Sobolev sense (after regularization) under suitable assumptions and with implications with regards to the optimality of strategies.

Is the Riemann Zeta Function in a Short Interval a 1-RSB Spin Glass?

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Warren Tai, *City University of New York, USA*

Fyodorov, Hiary & Keating established an intriguing connection between the maxima of log-correlated processes and the ones of the Riemann zeta function on a short interval of the critical line. In particular, they suggest that the analogue of the free energy of the Riemann zeta function is identical to the one of the Random Energy Model in spin glasses. In this talk, the connection between spin glasses and the Riemann zeta function is explored further. We study a random model of the Riemann zeta function and show that its two-overlap distribution corresponds to the one of a one-step replica symmetry breaking (1-RSB) spin glass. This provides evidence that the local maxima of the zeta function are strongly clustered.

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References


A \(O(\log n)\)-optimal policy for the dynamic and stochastic knapsack problem with equal values

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We study a dynamic and stochastic knapsack problem in which a decision maker is sequentially presented with \(n\) items and needs to select which items to include in a knapsack with fixed capacity \(c\). Arriving items have non-negative, independent sizes with common continuous distribution \(F\), and the decision maker needs to decide whether to select or reject an item when it is first presented and its size is revealed. The decision maker seeks to maximize the expected number of selected items, subject to the capacity constraint. We propose a simple adaptive online policy and prove that under mild regularity conditions on the distribution function \(F\), the expected number of selections of our heuristic policy is within \(O(\log n)\) of the optimal. We also discuss how the distribution of the number of selected items under such an adaptive policy compares with the number of items selected by the optimal policy.

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Metastability in condensing zero-range processes

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We consider zero-range processes with jump rates that decrease with the occupation number, which are known to exhibit a condensation phenomenon where a fraction of all particles concentrates on a single lattice site. We derive a scaling limit for the asymptotic stationary dynamics of the condensate location in the thermodynamic limit. Joint work with S. Grosskinsky and M. Loulakis.

References

Quenched asymptotics for the discrete fourier transforms of a stationary process

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This talk will present the results developed in the papers [1], [2], and [3], with emphasis in the technical aspects of [3], which is due to appear this year in Bernoulli. The results in question are rooted in the contributions presented in [4] and [5].

The framework is the following: let \((\Omega, \mathcal{F}, \mathbb{P})\) be a probability space, let \(T\) be an ergodic, invertible, bimeasurable transformation on \(\Omega\), let \(\mathcal{F}_0 \subset \mathcal{F}\) be a \(T\)-adapted sigma algebra: 

\[
A \in \mathcal{F}_0 \Rightarrow TA \in \mathcal{F}_0,
\]

and assume that \(E[|F_0|]\) is regular: there exists a decomposition \(E_0 := E[|F_0|]\) in the sense that for every fixed version of \(X \in L^1\)

\[
\omega \mapsto \int_{\Omega} X(z) d\mathbb{P}(z)
\]

defines an \(\mathcal{F}_0\)-measurable version of \(E_0 X\). A typical example of this setting is given by the case in which \(\Omega = \mathbb{R}^Z\) and \(\mathcal{F}_0 := \sigma((\xi_k)_{k \leq 0})\) where, for every \(k \in \mathbb{Z}\), \(\xi_k : \Omega \to \mathbb{R}\) is the projection on the \(k\)-th coordinate and \(T\) is the left shift, under the assumption that \((\xi_k)_{k \in \mathbb{Z}}\) is an ergodic (in particular, stationary) Markov chain.

In this setting, we address the following question: given an \(\mathcal{F}_0\)-measurable function \(X_0 \in L^2\) with \(E X_0 = 0\), what is the asymptotic behavior of the normalized discrete Fourier transforms

\[
\sqrt{n} S_n(\theta, \omega) = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} e^{ik\theta} X_0 \circ T^k(\omega)
\]

and of the corresponding cadlag functions \(t \mapsto \sqrt{n} S_{[nt]}(\theta, \omega), \) as \(n \to \infty\), under the measures \(\{\mathbb{P}_\omega\}_{\omega \in \Omega}\)?

We will remind how, after randomly recentering (i.e., considering \(\sqrt{n} (S_n(\theta, \omega) - E_0 S_n(\theta, \omega))\)), the Central Limit Theorem holds for a.e. \(\theta\) and \(\mathbb{P}_\omega\) without additional assumptions: this was developed in [2]. The necessity of the random centering \(\sim E_0 S_n(\theta, \omega)\)” will be discussed briefly as well, following [1].

But for the major part, the talk will be devoted to the problem of the asymptotics (as \(n \to \infty\)) of the random function \(t \mapsto \sqrt{n} S_{[nt]}(\theta, \omega)\). We will present a series of positive and negative results about this problem (as explained in [3]), and we will discuss some of the ideas behind the respective proofs. This will lead us in particular to present a few facts regarding the general theory of convergence in distribution and the pointwise ergodic theorem that apparently had not been noticed before.

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References


Sparse covariance estimation in high-dimensional deconvolution

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The estimation of the covariance matrix $\Sigma$ for a $p$-dimensional normal random vector is studied based on $n$ independent observations which are corrupted by additional noise. Only a very general non-parametric assumption is imposed on the distribution of the noise without any sparsity constraints on the errors covariance matrix. In this high-dimensional semi-parametric deconvolution problem spectral thresholding estimators are constructed that adapt to sparsity in $\Sigma$. We prove minimax convergence rates being logarithmic in $\log(p/n)$. The finite sample performance of the threshold estimators is illustrated in a numerical example.

Stochastic differential equations associated with coupled nonlinear reaction-advection-diffusion systems

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Nonlinear reaction-advection-diffusion systems arise as mathematical models in a number of applications such as magnetohydrodynamics, chemical kinetics, combustion, transport in porous media, biology and many others. We derive stochastic representations of the Cauchy problem generalized solutions of these systems which has the form

$$v^l_t + \sum_{m=1}^d c^l_m(v)\partial_x v^m = \sum_{m=1}^d \partial_x (a^l_m(v)\partial_x v^m) + f^l(v), \quad v^l(0, x) = v_0(x), t > 0, x \in \mathbb{R},$$

assuming nonlinearities $a(v), c(v) \in \mathbb{R}^{d \times d}, f(v) \in \mathbb{R}^d$ and initial data $v_0$ to be smooth smooth.

To explain our approach we consider one of the simplest versions of this type systems given by the so called MHD-Burgers system [1]

$$u_t + \frac{1}{2}(u^2 + B^2)_x = \frac{\mu^2}{2} u_{xx}, \quad u(0, x) = u_0(x), \quad (1)$$

$$B_t + (uB)_x = \frac{\sigma^2}{2} B_{xx}, \quad B(0, x) = B_0(x), \quad (2)$$

where $u$ and $B$ are the fluid velocity and magnetic field respectively, $\mu^2$ and $\sigma^2$ are constant diffusivities. This system is the simplest version to describe hydrodynamics in magnetic field with pressure generated by the magnetic field satisfying (2).

We prove the following statement.

**Theorem.** Assume that there exists a generalized (distributional) solution $u(t), B(t)$ of (1), (2) belonging to the Sobolev space $H_1$. Then this solution admits a probabilistic representation in the form

$$u(t, x) = E[u^l(t)u_0(\xi^l_{0, x}(t))], \quad B(t, x) = E[B^2(t)B_0(\hat{\xi}_{0, x}(t))], \quad (3)$$

where $\xi_k(\theta), \eta_k(\theta), k = 1, 2$ solve SDEs

$$d\xi_1(\theta) = \frac{1}{2} u(\theta, \xi_1(\theta))d\theta + \mu dw(\theta), \quad \xi_1(0) = x,$$

$$d\xi_2(\theta) = u(\theta, \xi_2(\theta))d\theta + \sigma dw(\theta), \quad \xi_2(0) = x,$$
\[ d\eta_1(\theta) = \frac{1}{2\mu} \left[ B(\theta, \xi_1(\theta)) \right]^2 \tilde{\eta}_1(\theta) dw(\theta), \quad d\eta_2(\theta) = 0, \quad \eta_1(0) = \eta_2(0) = 1, \]

and \( \tilde{\xi}_k(\theta), \tilde{\eta}_k(\theta) \) are time reversal processes with respect to \( \xi_k(\theta), \eta_k(\theta), k = 1, 2 \).

It should be noticed that the SDE system for processes \( \tilde{\xi}_k(\theta), \tilde{\eta}_k(\theta) \) with coefficients depending on unknown functions \( u(t, x), B(t, x) \) defined by (3) is not a closed one and to make it closed we need a probabilistic representation for derivatives \( u_x, B_x \) as well. Nevertheless we can derive a closed stochastic problem using results from [2], [3] and obtain in this way an independent probabilistic model for MHD-Burgers.

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**References**


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**Universality of height fluctuations for the dimer model**

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Benoit Laslier *Université Paris Diderot, France*

Gourab Ray *University of Cambridge, United Kingdom*

I will discuss a series of recent results with Benoit Laslier and Gourab Ray where we establish the convergence as the mesh size tends to zero of the centered height function associated to dimer models to a universal scaling limit (independent of the underlying microscopic details of the graph) in various situations. This includes in particular the cases of the hexagonal lattice in simply connected domains of the plane with planar boundary conditions of a fixed slope, and Temperleyan graphs drawn on Riemann surfaces. In the latter case, the height function consists of a one-form and an “instanton” component, both of which are shown to converge to a universal and conformally invariant limit. This generalises results by Dubédat (who proved this in the particular case of the torus and under an assumption of isoradiality), and solves some conjectures raised by his work. The proof relies on the “imaginary geometry” couplings between Gaussian free field and SLE curves as well as a nontrivial extension of Temperley’s bijection to Riemann surfaces. As an intermediate step of our proof we generalise and solve some questions raised by results of Kassel and Kenyon on the convergence of cycle-rooted spanning forests.

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A “thermodynamic” characterisation of some regularity structures near the subcriticality threshold

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We are interested in nonlinear stochastic partial differential equations with fractional Laplacian of order $\rho \in (0, 2)$ and driven by space-time white noise. Martin Hairer’s theory of regularity structures can be applied to these equations when they are locally subcritical, which requires $\rho$ to be larger than a critical value depending on the space dimension and the order of the nonlinear term. As $\rho$ approaches its critical value from above, the size of the negative-homogeneous sector of the model space diverges in a specific way, which can be determined by counting decorated rooted trees satisfying a number of constraints. We use tools from enumerative combinatorics and statistical physics to characterise the exponential growth of the number of trees and their degree distribution.

References


Metastability for interacting particles in double-well potentials and Allen-Cahn SPDEs

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We consider systems of overdamped particles in double-well potentials, with nearest neighbour coupling and subjected to weak noise. While for weak coupling the system behaves like an Ising model, for strong coupling it converges, in an appropriate scaling limit, to an Allen-Cahn SPDE. We obtain sharp asymptotics for metastable transition times of these systems. They follow an Arrhenius law, with exponent given by large deviation theory, with an Eyring-Kramers prefactor that can be determined by a potential-theoretic approach. In dimension 1 (joint work with Barbara Gentz) the prefactor is given by a Fredholm determinant. In dimension 2 (joint work with Giacomo Di Ges and Hendrik Weber) the equation needs to be renormalised, and the prefactor is given by a Carleman-Fredholm determinant.

The stable Lévy forest is the scaling limit of multitype Galton-Watson forests.

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Multitype Galton-Watson (GW) processes arise as a natural generalization of usual GW processes, in which individuals are differentiated by types that determine their offspring distribution. In this talk, we investigate the
ancestor trees and forests associated with irreducible multitype GW processes, when the total number of types is finite. Under criticality hypotheses on the mean matrix, and such that the offspring distributions belong to the domain of attraction of a stable law, we show that these forests (after a proper rescaling) converge to the continuum random stable forest.

Acknowledgement: This work is supported by the Swiss National Science Foundation 200021_144325/1

References


Convergence of delay differential equations driven by a fractional Brownian Motion for $1/3 < h < 1/2$

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We consider stochastic differential delay equations with hereditary drift driven by a fractional Brownian motion with Hurst parameter $1/3 < H < 1/2$. We show that, when the delay goes to zero, the solutions to these equations converge to the solutions for the equation without delay. The stochastic integral with respect to the fractional Brownian motion can be expressed as a Lebesgue integral using the fractional derivative for the case $H > 1/2$. We use the extension proposed by Hu and Nualart for the case where $1/3 < H < 1/2$.

Weak symmetric integrals with respect to the fractional Brownian motion

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We establish the weak convergence, in the topology of the Skorohod space, of the $\omega$-symmetric Riemann sums for functionals of the fractional Brownian motion when the Hurst parameter takes a critical value $H$ that depends on the symmetric measure $\nu$. As a consequence, we derive a change-of-variable formula in distribution, where the correction term is a stochastic integral with respect to a Brownian motion that is independent of the fractional Brownian motion. This is a joint work with I. Nourdin and D. Nualart.

On limit order books modelling

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A limit order book is a record of currently existing buy and sell orders for shares of a particular stock at a stock exchange. Mathematical modelling of the evolution of such records is of practical interest and is an interesting popular topic in modern applied probability theory.
We present some results of the empirical data analysis of the evolution of a real-life limit order book, for major company whose stock is traded at the Australian stock exchange (after preprocessing, the data included about 22 million limit and market orders placed over a year). Taking into account these findings, we construct a few zero intelligence Markovian models for the dynamics of limit order books and study their dynamics. In particular, we address the questions about ergodicity of the proposed models and the stock price evolution induced by the models dynamics.

On heat kernel decay for the random conductance model

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We study discrete time random walks in an environment of i.i.d. non-negative bounded conductances in $\mathbb{Z}^d$. We are interested in the anomaly of the heat kernel decay. We improve recent results and techniques. Explicitely, we prove the following:

Theorem. Let $d \geq 2$ and $\alpha > 0$. Consider a random walk in a random environment on the grid $\mathbb{Z}^d$ governed by bounded i.i.d. random conductances $(\omega_e)$ such that $P(\omega_e \in [0,1]) = 1$ and $P(\omega_e > 0)$ is larger than the critical value for Bernoulli bond percolation on $\mathbb{Z}^d$ and such that

$$(4d - 2) \lim_{u \to 0} \frac{\log P(\omega_e \in [u, 2u])}{\log u} < \alpha,$$

(C)

Then, there exists $c > 0$, such that for a.e. environment such that the origin $o$ belongs to the infinite cluster along positive conductances, for any $n$ large enough, we have

$$P_{o,o}^{2^n}(o,o) \geq c \pi(o) n^{-2+\alpha(d-1)}$$

(2.1)

Acknowledgement: I would like to thank Pierre Mathieu, Nina Gantert and Noam Berger for discussions.

References

On the extinction of lower hessenberg branching processes with countably many types

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Sophie Hautphenne The University of Melbourne, Australia, and EPFL, Switzerland

We consider the extinction events of a class of branching processes with countably many types which we refer to as Lower Hessenberg branching processes (LHBPs). These are multitype Galton-Watson processes with typeset $X = \{0, 1, 2, \ldots\}$, in which individuals of type $i$ may give birth to offspring of type $j \leq i + 1$ only. For this class of processes, we study the set $S$ of fixed points of the progeny generating function. In particular, we highlight the existence of a continuum of fixed points whose minimum is the global extinction probability vector $q$ and whose maximum is the partial extinction probability vector $\tilde{q}$. Our approach involves embedding a single-type explosive Galton-Watson process in a varying environment in the original LHBP and then establishing asymptotic relationships between the two processes.

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Kolmogorov equations and weak order analysis for SPDEs with multiplicative noise

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We consider the analysis of weak rates of convergence of numerical approximation for parabolic, semilinear, SPDEs, of the following form

$$dX_t = AX_t dt + G(X_t) dt + \sigma(X_t) dW_t,$$

where $W$ is a cylindrical Wiener process on $H = L^2(0, 1)$, $A$ is a linear operator, typically the Laplace operator with Dirichlet boundary conditions.

The abstract framework above includes stochastic reaction-diffusion and Burgers-type equations.

Consider $(X_n^T)_{n=0,1,...}$ the approximation obtained using the linear-implicit Euler scheme, with time-step size $\delta t$.

We are interested in the rate of convergence of the weak error

$$\mathbb{E}[^{\varphi}(X_T)] - \mathbb{E}[^{\varphi}(X_N^T)])$$

when $\delta t \to 0$, for a fixed $T = N\delta t > 0$, where $\varphi$ is a sufficiently regular real-valued function.

We generalize the approach based on the decomposition of the weak error using the solution $u(t, x) = \mathbb{E}_x[\varphi(X_t)]$ of the associated Kolmogorov equation

$$\frac{\partial u}{\partial t} = (Ax + G(x), Du(t, x)) + \frac{1}{2} \text{Tr}(\sigma(x)\sigma(x)^* D^2 u(t, x)).$$

The difficulty when $\sigma$ is not constant is to justify the well-posedness of all the terms in the right-hand side of the equation above, and to prove appropriate regularity bounds on $Du$ and $D^2 u$ to get the same rate of convergence as in the additive noise case ($\sigma$ constant).
I will first explain how to obtain the required regularity results for $Du$ and $D^2u$. Our approach is original: it uses a new family of two-sided stochastic integrals.
Then these regularity results are applied to prove that the weak error convergence converges with rate $1/2$.

References


Resource dependent branching processes and equilibria under immigration

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The objective of this talk is to create a framework in order to tackle questions of equilibria in human populations with immigration. We shall show that a model built around so-called resource dependent branching processes can cope well with this task, and we will derive the corresponding equilibrium equations. Our method of attack is based on the so-called Theorem of Envelopment (Bruss and Duerinckx, 2015), on several extinction criteria for bounded branching processes initiated by Zubkov (1970), as well as on inequalities for stopped sums of order statistics (Bruss and Robertson, 1991, and Steele, 2016). The obtained equilibrium equations are “delicate,” and as far as the speaker is aware, these results are new. Given the recent major changes of migration rates in many countries, the implications of these results seem relevant.

References


On the number of nodal domains of toral eigenfunctions

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We study the number of nodal domains of toral Laplace eigenfunctions. We establish a precise asymptotic result for “generic” eigenfunctions, which in particular implies an optimal lower bound for the number of nodal domains of...
a “typical” eigenfunction. Our proof combines Nazarov-Sodin’s results for nodal counts of random fields, Bourgain’s
de-randomisation procedure and a result of Bombieri-Bourgain on integer lattice points lying on a circle.

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Scaling Limits for Large Stochastic Networks

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We discuss several models for large stochastic networks given as weakly interacting pure jump (controlled) Markov
processes. Under suitable scaling we establish diffusion approximations, limits of stochastic control problems, and of
many-player stochastic dynamic games. Some features of limit models include infinite dimensional state descriptors,
degenerate diffusions, and mean field games for reflected processes. Based on joint works with Erhan Bayraktar,
Asaf Cohen and Eric Friedlander.

Conditional measures of determinantal point processes: the Gibbs
property and the Lyons–Peres Conjecture

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How do determinantal point processes behave under conditioning with respect to fixing the configuration in a
subset of the phase space? The talk will first address this question for specific examples such as the sine-process
or the process with the Bessel kernel, where one can explicitly write the analogue of the Gibbs property in our
situation. We will then consider processes induced by general self-adjoint kernels, for which, in joint work with Yanqi
Qiu and Alexander Shamov, it is shown that conditional measures of such processes are themselves determinantal
and governed by self-adjoint kernels, that the tail sigma-algebra for such process is trivial (a result independently
and by a completely different method obtained by Osada–Osada) and proof is given of the Lyons-Peres conjecture on
completeness of the system of kernels sampled at the particles of a random configuration. The talk is based on the

Hall-Littlewood processes and stochastic vertex models

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I will discuss recently found connections between vertex models (such as six-vertex/square ice model) and the
theory of symmetric functions (in particular, Hall-Littlewood functions). There are two types of applications. On a
probabilistic side, these connections allow to analyze the asymptotic behavior of certain vertex models (as well as their
degenerations like the asymptotic simple exclusion process) with the use of explicit formulas for their observables.
On a combinatorial side, we obtain a natural generalization of a Robinson-Shensted-Knuth algorithm.

Acknowledgement: Based on joint works with A. Borodin, K. Matveev, M. Wheeler.
Bayesian test for multi-channel signal detection problem

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We consider a problem of detection a signal with unknown energy in a multi-channel system, containing a big number of channels. We assume that the signal can appear in the k-th channel with a known small prior probability \( \pi_k \). Using observations from all channels we would like to detect whether the signal is presented in one of the channels or we observe pure noise. In our work we describe and compare statistical properties of maximum posterior probability test and optimal Bayesian test. In particular, for these tests we obtain limiting distributions of test statistics and define sets of undetectable signals.

Acknowledgement: the research was supported by the Russian Science Foundation grant (project 14-50-00150).

Regularization by noise and flows of solutions for a stochastic heat equation

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Motivated by the regularization by noise phenomenon for SDEs we prove existence and uniqueness of the flow of solutions for the non-Lipschitz stochastic heat equation

\[
\frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial z^2} + b(u(t, z)) + \dot{W}(t, z), \quad t \geq 0, z \in \mathbb{R}
\]

where \( \dot{W} \) is a space-time white noise on \( \mathbb{R}_+ \times \mathbb{R} \) and \( b \) is a bounded measurable function on \( \mathbb{R} \). As a byproduct of our proof we also establish the so-called path-by-path uniqueness for any initial condition in a certain class on the same set of probability one. This extends recent results of Davie (2007) to the context of stochastic partial differential equations. Some results concerning distributional drifts will also be discussed.

References


Scaling limits and other results on the seed bank model

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In the last decades, population models which are generalizations or variants of the classical Wright-Fisher model have been thoroughly analyzed. In this talk, we explore the (biallelic) Wright-Fisher diffusion with seed bank ([1]) comparing it, among others, to the structured Wright-Fisher diffusion with two islands ([2]; [3]). Despite their seeming similarity, both systems exhibit remarkable qualitative differences. We illustrate this by an analysis of various aspects such as moments, stationary distribution, reversibility, sample heterozygosity, sampling formulas, and scaling limits. One of the main used tools is duality. In particular, in we calculate the limit for the migration rate going to zero for a fast time scale we obtain a new object called the rare ancient types process.

Based on joint work with Prof. Dr. Blath (TU), Dr. González Casanova (WIAS Berlin) and Dr. Wilke Berenguer (TU).

Acknowledgement: Berlin Mathematical School, RTG 1845.

References


Local large deviations and the strong renewal theorem

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We establish two different, but related results for random walks in the domain of attraction of a stable law of index $\alpha$. The first result is a local large deviation upper bound, valid for $\alpha \in (0, 1) \cup (1, 2)$, which improves on the classical Gnedenko and Stone local limit theorems. The second result, valid for $\alpha \in (0, 1)$, is the derivation of necessary and sufficient conditions for the random walk to satisfy the strong renewal theorem (SRT). This solves a long standing problem, which dates back to the 1963 paper of Garsia and Lamperti for renewal processes (i.e. random walks with non-negative increments), and to the 1968 paper of Williamson for general random walks.
Mean Field Stackelberg Games with Heterogeneous Followers

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We consider general Mean Field Stackelberg Games with Lipschitz and continuously differentiable coefficient. Each follower is subject to a delay impact from the leader. Necessary conditions for the optimal controls for both the leader and followers are given by a set of six equations. A comprehensive study on the linear quadratic case is given. We provide a set of time-independent sufficient conditions, which guarantee the existence of a unique solution. Several numerical results are demonstrated.

Acknowledgement: The presenter acknowledges the financial support from Imperial College London and The University of Hong Kong.

References


Regularization by noise for a class of McKean-Vlasov stochastic differential equations

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In a seminal work of Zvonkin in 1974, it is shown that infinitesimal stochastic perturbation can restore uniqueness for ordinary differential system. In this talk, we show that this phenomenon still occurs for a class of McKean-Vlasov stochastic differential equations with rough drift in space and measure arguments. This last result could seems unexpected at first sight, since the noise only acts on the space variable. We will show that some particular structural assumptions on the system allow us to recover this phenomenon.

Our proof, in the spirit of Zvonkin Work, consists in exhibiting suitable smoothing properties of the Partial Differential Equation (PDE) associated to the generator of the Markov Process. This leads us to study a PDE set on the Cartesian product of the real and the probability measure on the real line, where the derivative along the measure is understood in the Lions sense. Especially, a smoothing effect in the measure direction is exhibited.
Some recent results in self-similar Markov processes

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Piotr Graczyk Université d’Angers, France
Tomasz Żak Wrocław University of Science and Technology, Poland

We show that any $\mathbb{R}^d \setminus \{0\}$-valued self-similar Markov process $X$, with index $\alpha > 0$ can be represented as a path transformation of some Markov additive process (MAP) $(\theta, \xi)$ in $S_{d-1} \times \mathbb{R}$. This result extends the well known Lamperti transformation. Let us denote by $\tilde{X}$ the self-similar Markov process which is obtained from the MAP $(\theta, \xi)$ through this extended Lamperti transformation. Then we prove that $\tilde{X}$ is in weak duality with $X$, with respect to the measure $\pi(x/\|x\|)\|x\|^{\alpha-d}dx$, if and only if $(\theta, \xi)$ is reversible with respect to the measure $\pi(ds)dx$, where $\pi(ds)$ is some $\sigma$-finite measure on $S_{d-1}$ and $dx$ is the Lebesgue measure on $\mathbb{R}$. Moreover, the dual process $\tilde{X}$ has the same law as the inversion $(X_t/\|X_t\|^2, t \geq 0)$ of $X$, where $\gamma_t$ is the inverse of $t \mapsto \int_0^t \|X_s\|^{-2\alpha} ds$. These results allow us to obtain excessive functions for some classes of self-similar Markov processes such as stable Lévy processes.

References


The favorite sites of subdiffusive biased walks on a Galton-Watson tree

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Erdős and Révész initiated the study of favorite sites by considering the one-dimensional simple random walk. We investigate in this paper the same problem for a class of null-recurrent randomly biased walks on a super-critical Galton-Watson tree. We prove that in the subdiffusive regime, there is some parameter $\kappa \in (1, \infty)$ such that the set of the favorite sites of the biased walk is almost surely bounded in the case $\kappa \in (2, \infty]$, tight in the case $\kappa = 2$, and oscillates between a neighborhood of the root and the boundary of the range in the case $\kappa \in (1, 2)$. The proof relies on the exploration of the Markov property of the local times process with respect to the space variable and on a precise tail estimate on the maximum of local times, using a change of measure for multi-type Galton-Watson trees. This is a joint work with Loïc de Raphaelis of Université Paris VI and Yueyun Hu of Université Paris XIII.
Characterization of pinched Ricci curvature by functional inequalities

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This talk is based on joint work with Anton Thalmaier. We discuss using functional inequalities for diffusion semigroups on Riemannian manifolds (possibly with boundary) to characterize the pinched Ricci curvature, along with gradient estimates, $L^p$-inequalities and log-Sobolev inequalities. These results are further extended to differential manifolds carrying geometric flows. As application, it is shown that they can be used in particular to characterize general geometric flow and Ricci flow by functional inequalities.

On the maximum of characteristic polynomials of random matrices

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Thomas Madaule Université Paul Sabatier, France
Joseph Najnudel University of Cincinnati, USA

I want to explain why characteristic polynomials of (finite) random matrices behave like log-correlated Gaussian fields. More precisely, they form a peculiar regularization of the so-called Gaussian Multiplicative Chaos, introduced by Kahane. The analogy leads to many conjectures on the fractal measure defined by absolute continuity with respect to the Lebesgue measure, or simply the global maxima.

Among the rigorous results, I will report on a recent work in collaboration with J. Najnudel and T. Madaule, where we made progress in understanding the extremal values of (the logarithm of) the characteristic polynomial of a random unitary matrix whose spectrum is distributed according the Circular Beta Ensemble (CβE).

Our result answers a conjecture of Fyodorov, Hiary and Keating, originally formulated for the $\beta = 2$ case, which corresponds to the CUE field. In the formulation of this conjecture, one recognizes the same ingredients as maxima of log-correlated processes, branching Brownian motion or branching random walks...

**Keywords:** Hierarchical structure, Extremas of log-correlated fields, Random Matrix Theory, Circular $\beta$ Ensembles, Orthogonal polynomials on the unit circle (OPUC).

References


Fluid limits with random initial conditions in density dependent populations with high carrying capacity

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Pavel CHIGANSKY \textit{The Hebrew University of Jerusalem, Israel}, E-mail: Pavel.Chigansky@mail.huji.ac.il
Fima C. Klebaner \textit{Monash University, Australia}

In this talk I will present a phenomenon occurring in population processes that start near zero and have large carrying capacity. As is well known, such processes, normalized by the carrying capacity, converge on finite intervals to the solutions of ordinary differential equations, also known as the fluid limit. When the initial population is small relative to carrying capacity, this limit is trivial. Here we show that, viewed at suitably chosen times increasing to infinity, the process converges to the fluid limit, governed by the same dynamics, but with a random initial condition. This random initial condition is related to the martingale limit of an associated linear birth and death process.

References


Intermittency and moment growth indices for stochastic PDEs driven by Lévy noise

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Péter Kevei \textit{University of Szeged, Hungary}

We investigate the behavior of moments at large times for the stochastic heat equation driven by multiplicative Lévy noise. Unlike the case of Gaussian noise, moments only exist up to the order $1 + 2/d$, where $d$ is the spatial dimension. Weak intermittency of order $p$, that is, the exponential growth of the $p$-th moment of the solution as time tends to infinity, is established for values of $p$ that are sufficiently close to the critical exponent $1 + 2/d$. Crucial to the proof is a new moment lower bound for stochastic integrals driven by compensated Poisson random measures. Moreover, again in contrast to the Gaussian case, we show that the exponential growth rates themselves increase at a superexponential speed as $p$ approaches $1 + 2/d$, indicating extreme fluctuations of the solution at large times. Results are reported for non-vanishing as well as for compactly supported initial values. (This is joint work with Péter Kevei, University of Szeged.)
Non-asymptotic results for multivariate statistics based on random size samples

Gerd CHRISTOPH Otto-von-Guericke University of Magdeburg, Germany, E-mail: gerd.christoph@ovgu.de
Michael Monakhov Lomonosov Moscow State University, Russia
Vladimir Ulyanov Lomonosov Moscow State University, Russia

For distribution of normalized random means based on samples with random size we give asymptotic expansions with Student or Laplace limit laws. Samples of random size and non-normal limit laws occur e.g. in insurance, economics, biology and for modeling city-size growth or high-frequency stock index returns, see [1], [2].

Let $X_1, X_2, \ldots$ be i. i. d. random variables (r.v.) with $E[|X_1|^3] < \infty$, $E(X_1) = \mu$, $0 < \text{Var}(X_1) = \sigma^2$, skewness $\lambda_3 = \sigma^3 \mathbb{E}(X_1 - \mu)^3$ and kurtosis $\lambda_4 = \sigma^4 \mathbb{E}(X_1 - \mu)^4$ and suppose that r.v. $X_1$ admits Cramér’s condition: \( \limsup_{|t| \to \infty} \left| \mathbb{E}[e^{itX_1}] \right| \leq 1 \). We denote the mean $T_m = (X_1 + \cdots + X_m) / m$, $m = 1, 2, \ldots$ Then one has
\[
\sup_x |\mathbb{P}(\sigma \sqrt{m}T_m - \mu \leq x) - \Phi_m(x)| \leq C_1 m^{-3/2},
\]
where $\Phi_m(x)$ is the second order Edgeworth expansion with normal limit law.

Consider now a random mean $T_{N_n}$ with a random sample size $N_n = N_n(r)$ of observations $X_1, X_2, \ldots$ and $N_n$ is independent of them, where $N_n(r)$ is a negative binomial distributed (shifted by 1) rv. such that
\[
\mathbb{P}(N_n(r) = i) = \Gamma(i + r - 1)(i - 1)! \Gamma(r)^{-1} (1 - 1/n)^{i - 1}, \quad r > 0, \quad i, n \in \mathbb{N} := \{1, 2, \ldots\}.
\]
Let $g(n) = \mathbb{E}(N_n(r)) = r(n - 1) + 1$, then $\mathbb{P}(N_n(r)/\mathbb{E}(N_n(r)) \leq x)$ tends to the gamma distribution $G_{r,r}(x)$ having density $g_r(x) = r^x x^{r-1} e^{-rx} / \Gamma(r)$ and the limit distribution of $\mathbb{P}\left( \sigma \sqrt{g(n)}(T_{N_n(r)} - \mu) \leq x \right)$ is the Student $t$-law $S_2(x)$ with density $s_2(x) = \Gamma(r + 1/2)(\sqrt{2\pi} \Gamma(r))^{-1} (1 + x^2/(2r))^{-(r+1)/2}$, see [1] or [2] and references therein.

**Lemma.** Suppose $r \geq 1$. For $x > 0$ and all $n \geq 2$ there exists a real number $C_2(r) > 0$ such that
\[
\sup_{x \geq 0} |\mathbb{P}(N_n(r) \leq g(n)x) - G_{r,r}(x) + g_r(x)((x - 1)(r - 2) - 2Q_1(g(n)x))/2r(n - 1)| \leq C_2(r) n^{-\min(r,2)},
\]
where $Q_1(y) = 1/2 - (y - [y])$ and $[y]$ is the integer part of $y$ with $y - 1 < [y] \leq y$.

Using Theorem 3.1 of [1] and the second order Edgeworth type expansions of $T_n$ and $N_n$ we get new expansion for the random mean $T_{N_n}(r)$ with $r > 1$, which allows also to obtain the Cornish-Fisher expansion, see [3]. For shortage of space the results are given only for $r = 2$.

**Theorem.** Let $r = 2$. Under the mentioned conditions there exists a constant $C > 0$ such that
\[
\sup_x |\mathbb{P}\left( \sigma \sqrt{2n} - T_{N_n} - \mu \leq x \right) - S_4(x) + \left( A_1(x) + A_2(x) \right) \left( 1 - \frac{1}{2n - 1} \right) | \leq C n^{-3/2},
\]
with $A_1(x) = \lambda_3 (x^2 - 2) / 9$ and $A_2(x) = x (10\lambda_3^2 / (9 (4 + x^2)) - \lambda_4 / 6)$.

Let $x_\alpha = u_\alpha - A_1(u_\alpha) / \sqrt{2n - 1} + \left( s_1(u_\alpha) \right) A_1(u_\alpha) - A_2(u_\alpha) / \sqrt{2n - 1} + \mathcal{O}(n^{-3/2})$, $n \to \infty$.

Similar results hold, when $N_n$ is the maximum of $n$ i.i.d. discrete Pareto r. v. In this case the Laplace law is the limit distribution of $T_{N_n}$. The extensions of the results to the multivariate statistics are discussed as well.

**References**


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Second order edgeworth and cornish-fisher expansions for statistics based on random means

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For distribution of normalized random means based on samples with random size we give asymptotic expansions with Student or Laplace limit laws. Samples of random size and non-normal limit laws occur e.g. in insurance, economics, biology and for modeling city-size growth or high-frequency stock index returns, see [1], [2].

Let $X_1, X_2, \ldots$ be i. i. d. random variables (r.v.) with $\mathbb{E}[X_1^6] < \infty$, $\mathbb{E}(X_1) = \mu$, $0 < \text{Var}(X_1) = \sigma^2$, skewness $\lambda_3 = \sigma^3 \mathbb{E}(X_1 - \mu)^3$ and kurtosis $\lambda_4 = \sigma^4 \mathbb{E}(X_1 - \mu)^4$ and suppose that r.v. $X_1$ admits Cramér’s condition: $\limsup_{t \to \infty} \left| \mathbb{E}e^{tX_1} \right| < 1$. We denote the mean $T_m = (X_1 + \cdots + X_m) / m$, $m = 1, 2, \ldots$. Then one has

$$\sup_x \left| \mathbb{P}(\sigma \sqrt{n}(T_m - \mu) \leq x) - \Phi_{m,2}(x) \right| \leq C_1 m^{-3/2},$$

where $\Phi_{m,2}(x)$ is the second order Edgeworth expansion with normal limit law.

Consider now a random mean $T_{N_n}$ with a random sample size $N_n = N_n(r)$ of observations $X_1, X_2, \ldots$ and $N_n$ is independent of them, where $N_n(r)$ is a negative binomial distributed (shifted by 1) r.v. such that

$$\mathbb{P}(N_n(r) = i) = \Gamma(i + r - 1)((i - 1)!)^r (1 - 1/n)^{i - 1}, \quad r > 0, \quad i, n \in \mathbb{N} := \{1, 2, \ldots\}.$$

Let $g(n) = \mathbb{E}(N_n(r)) = r(n - 1) + 1$, then $\mathbb{P}(N_n(r)/\mathbb{E}(N_n(r)) \leq x)$ tends to the gamma distribution $G_{r,r}(x)$ having density $g_{r,r}(x) = r^x x^{-1} e^{-rx} / \Gamma(r)$ and the limit distribution of $\mathbb{P}\left( \sigma \sqrt{g(n)}(T_{N_n(r)} - \mu) \leq x \right)$ is the Student $t$ - law $S_{2r}(x)$ with density $s_{2r}(x) = \Gamma(r + 1/2)(\sqrt{2\pi} \Gamma(r))^{-1} \left( 1 + x^2/(2r) \right)^{-(r+1/2)}$, see [1] or [2] and references therein.

**Lemma.** Suppose $r \geq 1$. For $x > 0$ and all $n \geq 2$ there exists a real number $C_2(r) > 0$ such that

$$\sup_{x \geq 0} \left| \mathbb{P}(N_n(r) \leq g(n)x) - G_{r,r}(x) + g_{r,r}(x)((x - 1)(r - 2) - 2Q_1(g(n)x)/(2r(n - 1))) \right| \leq C_2(r) n^{-\min\{r,2\}},$$

where $Q_1(y) = 1/2 - (y - [y])$ and $[y]$ is the integer part of $y$ with $y - 1 < [y] \leq y$.

Using Theorem 3.1 of [1] and the second order Edgeworth type expansions of $T_m$ and $N_n(r)$ we get new expansion for the random mean $T_{N_n}(r)$ with $r > 1$, which allows also to obtain the Cornish-Fisher expansion, see [3]. For shortage of space the results are given only for $r = 2$.

**Theorem.** Let $r = 2$. Under the mentioned conditions there exists a constant $C > 0$ such that

$$\sup_x \left| \mathbb{P}\left( \sigma \sqrt{2n - 1}(T_{N_n} - \mu) \leq x \right) - S_4(x) + \left( A_1(x) \frac{1}{\sqrt{2n - 1}} + A_2(x) \frac{1}{2n - 1} \right) s_4(x) \right| \leq C n^{-3/2},$$

with $A_1(x) = \lambda_3 (x^2 - 2) / 9$ and $A_2(x) = x (10\lambda^3_3/9 (4 + x^2)) - \lambda^6_4 / 6$.

Let $x_\alpha$ and $u_\alpha$ be $\alpha$-quantiles of $\sigma \sqrt{g(n)}(T_{N_n} - \mu)$ and the Student $t$-distribution $S_4(x)$, respectively. Then

$$x_\alpha = u_\alpha - A_1(u_\alpha) \frac{1}{\sqrt{2n - 1}} + \left( A'_1(u_\alpha) A_4(u_\alpha) A_1(u_\alpha) - A_2(u_\alpha) \right) \frac{1}{2n - 1} + O(n^{-3/2}), \quad n \to \infty.$$

Similar results are obtained for the case, when the random size $N_n$ is the maximum of $n$ i.i.d. discrete Pareto r. v. In this case the Laplace law is the limit distribution of the normalized random mean $T_{N_n}$. 

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References


Sailboat Trajectory Optimization

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We study the optimal strategy for a sailboat to reach an upwind island under the hypothesis that the wind direction fluctuates according to a Brownian motion. The problem is singular because we assume that there is no loss of time when tacking. We exhibit the optimal strategy. The proof of optimality, since the HJB equation does not admit a closed form solution, involves an intricate estimate of derivatives of the value function. Finally we explicitly provide the asymptotic behavior of the value function and we give some new insights on the stochastic flow of a reflected SDE.

Concentration inequalities for Harris recurrent Markov chains

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Concentration inequalities are a powerful tool to control the tail probability that a random variable $X$ exceeds some prescribed value $t$. They are a crucial step in deriving many results in numerous fields such as statistics, learning theory, discrete mathematics, statistical mechanics, information theory or convex geometry. The purpose of this talk is to present Bernstein type inequality for unbounded classes of functions $F$ and Hoeffding type functional inequality for Harris recurrent Markov chains. To avoid some complicated mixing conditions, we make use of the well-known regeneration properties of Markov chains. It is noteworthy that when deriving exponential inequalities for Markov chains (or any other process with some dependence structure) one can not expect to recover fully the classical results from the i.i.d. case. The goal is then to get some counterparts of the inequalities for i.i.d. random variables with some extra terms that appear in the bound as a consequence of a Markovian structure of the considered process. Our inequalities allow to obtain fast rates of convergence in mathematical statistics. Moreover, all constants involved in our bounds of the considered inequalities are given in an explicit form which can be advantageous in practical considerations. Firstly, we present the theory for regenerative Markov chains, next we show how to generalize these results and establish exponential bounds for Harris recurrent case.

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Intermittency properties for a family of SPDEs driven by a fractional-type noise

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In this talk, we will establish intermittency properties for a family of Stochastic Partial Differential Equations driven by a multiplicative Gaussian noise that has similar asymptotic properties as fractional Brownian motion in time and (possibly) some covariance in space. The solutions exhibit similar aspects as the solution to the SPDEs driven by (standard) fractional noise, yet our noise allows to work with Walsh integration techniques and can thus be studied with less technical tools. We will illustrate in what aspects the noise and the solutions are similar and in what aspects they are different from regular fractional noise. Examples include the stochastic heat and wave equations.

Acknowledgement: Daniel Conus’s research is partially funded by NSF Grant DMS-1513556.

Existence and uniqueness of obliquely reflecting diffusions in cusps

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In their 2009 paper [5], Kang, Kelly, Lee and Williams propose a diffusion approximation for the workload process in a model for a network operating under a weighted α-fair bandwidth sharing policy. For α = 1 the limit is an obliquely reflecting diffusion in a convex polyhedron. For α ≠ 1 the state space is not a convex polyhedron and the diffusion approximation cannot be proved, specifically due to lack of suitable (weak) existence and uniqueness results for the limiting reflecting diffusion. In particular, for α < 1 the state space presents cusp-like singularities.

To the knowledge of the authors, up to today the only existence and uniqueness results for semimartingale reflecting diffusions in domains with cusp-like singularities are the results by De Blassie and Toby [3], [4] for Brownian motion in 2-dimensional cusps of the form $-x_1^d < x_2 < x_1^c$ with direction of reflection on each side that forms a constant angle with respect to the normal (which is not the case in the problem posed in [5]). Burdzy and Toby [1] and Burdzy, Kang and Ramanan [2] have studied the case of constant, vertical directions of reflection, when the reflecting Brownian motion is not a semimartingale.

We prove weak existence and uniqueness for a general semimartingale reflecting diffusion with general directions of reflection in a 2-dimensional cusp of the same form as De Blassie and Toby, under the assumption that, at the origin,
there exists a vector in the right half space that has positive scalar product with both the directions of reflection. This should be an intermediate step towards addressing the problem posed in [5], which is in higher dimension.

References


The Toom Interface via Coupling

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We consider an interesting one dimensional interacting particle system related to a famous probabilistic cellular automata called Toom’s model. The state space of this particle system may be taken to be either $\{-1, 1\}^Z$ or $\{-1, 1\}^N$ and the dynamics is defined such that there is a single parameter, call it $\lambda$, controlling the relative rate at which the spins change their states ($\lambda = 1/2$ being the unbiased case). The particle system has a number of remarkable features. First and foremost, when defined on $N$, it has a unique invariant measure for each choice of $\lambda$. Moreover, if one asks about the scaling of the variance of the first $L$ spins with $L$, it has been conjectured to scale as $L^{2/3}$ if $\lambda$ is not 1/2 and $L^{1/2}$ up to logarithmic corrections if $\lambda = 1/2$ (recall that for independent spins, the scaling is $L$). These exponents are thought to directly reflect the fact that the dynamical behavior of fluctuations is governed by either the Kardar-Parisi-Zhang equation (if $\lambda$ is not 1/2) or by the stochastic heat equation with a few caveats (if $\lambda = 1/2$). Inspired by this picture, in the talk I will detail an investigation into the basic properties enjoyed by Toom interfaces.

References

The infinitely many zeros of stochastic coupled oscillators driven by random forces

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In this work, previous results concerning the infinitely many zeros of single stochastic oscillators driven by random forces are extended to the general class of coupled stochastic oscillators. We focus on three main subjects: 1) the analysis of this oscillatory behavior for the case of coupled harmonic oscillators; 2) the identification of some classes of coupled nonlinear oscillators showing this oscillatory dynamics and 3) the capability of some numerical integrators - thought as discrete dynamical systems - for reproducing the infinitely many zeros of coupled harmonic oscillators driven by random forces.

Multi-refracted and level-dependent Lévy risk processes

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We consider multi-refracted Lévy risk process whose dynamics change by subtracting a fixed linear drifts whenever the process is above certain levels. Formally, we define a multi-refracted Lévy risk process as a unique strong solution of the SDE (for $k \geq 1$):

$$dU_k(t) = dX(t) - \left( \delta_1 \mathbf{1}_{\{U_k(t) > b_1\}} + \delta_2 \mathbf{1}_{\{U_k(t) > b_2\}} + \ldots + \delta_k \mathbf{1}_{\{U_k(t) > b_k\}} \right) dt, \quad t \geq 0,$$

where $X$ is a spectrally negative Lévy process, $\delta_1,\ldots,\delta_k \in \mathbb{R}$ and $b_1 < b_2 < \ldots < b_k$ are model parameters.

Moreover, we present the formulas for one and two sided exit problems written in terms of the new $q$-scale functions associate with the process $U_k$. We also present new properties of the obtained scale functions. Finally, we extend the theory of multi-refracted processes to processes with general premium rate function $\phi$.

This talk is based on joint work with Tomasz Rolski, José-Luis Pérez and Kazutoshi Yamazaki.

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Asymptotics of self-similar growth-fragmentation processes

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Markovian growth-fragmentation processes introduced in [2,3] extend the pure-fragmentation model by allowing the fragments to grow larger or smaller between dislocation events. What becomes of the known asymptotic behaviors of self-similar pure fragmentations [1,5,6,7] when growth is added to the fragments is a natural question that we investigate in this paper. Our results involve the terminal value of some additive martingales whose uniform integrability is an essential requirement. Dwelling first on the homogeneous case [2], we exploit the connection with branching random walks and in particular the martingale convergence of Biggins [8,9] to derive precise asymptotic estimates. The self-similar case [3] is treated in a second part; under the so called Malthusian hypotheses and with
the help of several martingale-flavored features recently developed in [4], we obtain limit theorems for empirical measures of the fragments.

References


Asymptotics of superpositions of stable point processes

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We interpret the definition of the Papangelou intensity as an integration by parts in the sense of Malliavin calculus for point processes. Mixed with the Stein-Malliavin method [1], we analyze the Kolmogorov-Rubinstein distance discrete α-stable processes [2] of different characteristics. Rate of convergence of superposition of stable processes is also given.

References


Renewal theorems for Markov chains

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We consider a one-dimensional transient Markov chains $X = (X_n)_{n \geq 0}$ on a positive half line. Let

$$H_y(x) = \sum_{n=0}^{\infty} P_y(X_n \leq x)$$

be the renewal function of $X$. In this talk I will discuss integral and local renewal theorems for $H_y(x)$, as $x \to \infty$. I will mainly consider $X$ with the decreasing vanishing drift coefficient

$$E_x[X_1 - x] \sim \frac{\mu}{x^{\beta}}, \ x \to \infty,$$

where $\beta \in (0, 1)$ and asymptotically constant diffusion coefficient

$$E_x[(X_1 - x)^2] \to b \in (0, \infty).$$

For example, for $\beta = 1$ and $2\mu > b$ the following integral renewal theorem can be obtained for a fixed initial state $y$,

$$H_y(x) \sim \frac{x^2}{2\mu - b}, \ x \to \infty.$$

The talk is based on [1] and [2].

References


A class of solutions of Backward Stochastic Differential Equations

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Under some restrictions of the time interval, we compare solution of a class of backward stochastic differential equations, backward stochastic Volterra integral equations and backward doubly stochastic differential equations with the solution of corresponding simpler equation of appropriate type; to be precise, the relations between their solutions under different conditions for coefficients on their generator functions is given. Using these relations, it could be easier to study solutions of more complex equations, where coefficients in backward integrals could be treated as perturbations.

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References


Unboundedness of Tsallis SDE: the role of overdamped approximation

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An apparently ideal way to generate continuous bounded stochastic processes is to consider the stochastically perturbed motion of a point of small mass in an infinite potential well, under overdamped approximation. Here, however, we show that the aforementioned procedure can be fallacious and lead to incorrect results. We indeed provide a counter-example concerning one of the most employed bounded noises, hereafter called Tsallis-Stariolo-Borland (TSB) noise, which admits the well known Tsallis q-statistics as stationary density. In fact, we show that for negative values of the Tsallis parameter q (corresponding to sufficiently large diffusion coefficient of the stochastic force), the motion resulting from the overdamped approximation is unbounded. We then investigate the cause of the failure of Kramers first type approximation, and we formally show that the solutions of the full Newtonian
non-approximated model are bounded, following the physical intuition. Finally, we provide a new family of bounded noises extending the TSB noise, the boundedness of whose solutions we formally show.

*Keywords:* Bounded noise; non-Gaussian noise; Tsallis q-statistics; Newton’s equation; Overdamped approximation; Potential well; Stochastic Differential Equations.

**References**


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**A New Approach to Sequential Stopping for Stochastic Simulation**

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**Peter Glynn** *Stanford University, USA*

Simulation is a powerful numerical tool set for performance evaluation and optimization of stochastic systems. Successful implementation of this numerical approximation scheme requires one being able to assess the quality of the estimators and control the estimation errors. In this talk, I will present a sequential stopping problem for a class of simulation problems in which variance estimation is difficult. We establish the asymptotic validity of sequential stopping procedures for estimators constructed using the sectioning (replication) methods with a fixed number of sections. The limiting distribution of the estimators at stopping times as the error size (the absolute error or the relative error) goes to 0 is characterized in closed form. This limiting distribution is different from the limiting distribution of the estimator constructed based on a fixed number of samples as the sample size goes to infinity, which indicates that we need a different scaling parameter when constructing the corresponding confidence intervals using the sequential stopping procedure. We also investigate the empirical performance of our proposed sequential stopping algorithms through some simulation experiments.
Multitype branching processes in random environment: the critical case

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We consider a \(p\)-type Galton-Watson branching process \(Z(n) = (Z^{(1)}(n), \ldots, Z^{(p)}(n))\), \(n = 0, 1, \ldots\), in i.i.d. random environment. Let \(M_n = (m_{ij}(n))_{i,j=1}^p\) be the mean matrix of the reproduction law for the particles of the \(n\)-th generation and \(\pi\) be the upper Lyapunov exponent of the sequence \(\{M_n, n = 0, 1, \ldots\}\) of i.i.d. random matrices. Suppose that there exists a number \(b > 1\) such that
\[
\frac{1}{b} \leq \frac{\max_{i,j} m_{ij}(n)}{\min_{k,l} m_{k,l}(n)} \leq b
\]
with probability 1. Let \(e_i, i = 1, \ldots, p\), be the \(p\)-dimensional vector whose \(i\)-th component is equal to 1 and the others are zeros, \(\mathbf{0} = (0, \ldots, 0)\) be the \(p\)-dimensional vector all whose components are zeros.

We show that if \(\pi = 0\), then, under some mild additional technical conditions, for any \(i \in \{1, \ldots, p\}\) there exists a number \(c_i \in (0, \infty)\) such that
\[
\lim_{n \to \infty} \frac{1}{\sqrt{n}} \mathbb{P}(Z(n) \neq \mathbf{0} | Z(0) = e_i) = c_i.
\] (1)

This result refines the main statements of [2] where (1) was proved under the assumption that the offspring generating functions of particles are fractional linear and where, for the general form of reproduction laws of particles, it was shown that for any \(i \in \{1, \ldots, p\}\) there exist positive constants \(c\) and \(C\) such that
\[
c \leq \frac{1}{\sqrt{n}} \mathbb{P}(Z(n) \neq \mathbf{0} | Z(0) = e_i) \leq C
\]
for all \(n \geq 1\).

Our result also complements the respective theorems from [3] and [4], where an asymptotic expression for the probability \(\mathbb{P}(Z(n) \neq \mathbf{0} | Z(0) = e_i)\) is found for the general form of the offspring distributions of particles under the assumption that matrices \(\{M_n, n = 0, 1, \ldots\}\) have with probability 1 a common nonrandom positive left or right eigenvector corresponding to their Perron roots.

References

Large deviation estimates for exceedance times of branching process in random environment

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We will concentrate on a population growth process in which the reproduction law is randomly picked at each generation. Denote by $Q = \{Q_n\}_{n \geq 0}$ a sequence of iid random measures on the set of non-negative integers $\mathbb{N}$. The sequence $\{Z_n\}_{n \geq 0}$ is called a branching process in random environment if $Z_0 = 1$, and

$$Z_{n+1} = \sum_{k=1}^{Z_n} \xi_k^n,$$

where given $Q$, $\xi_k^n$ are iid and independent from $Z_n$ with common distribution $Q_n$. Our first goal is to establish a precise large deviation estimates, i.e. to find a precise asymptotic of

$$P[Z_n > e^{\rho n}] \quad \text{as} \quad n \to \infty,$$

where $\rho > 0$ is a fixed parameter. Further investigation of our techniques reveals that apart form knowing the probability of large deviations of $\{Z_n\}_{n \geq 0}$, we can also point out the moment when it arises. More precisely, we will consider the exceedance time (or first passage time) for $\{Z_n\}_{n \geq 0}$ defined viz.

$$T_t = \inf\{n \geq 0 \mid Z_n > t\}.$$

We will study the large deviations of $\{T_t\}_{t \geq 0}$ and describe the probability that $\{Z_n\}_{n \geq 0}$ exceeds some threshold $t$ precisely at some given moment, that is we will find the asymptotic of

$$P \left[ T_t = \left\lfloor \frac{\log(t)}{\rho} \right\rfloor \right] \quad \text{as} \quad t \to \infty,$$

where again $\rho > 0$ is some fixed parameter.
Infinite Lévy particles with long range interactions

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In this talk, we study infinite dimensional stochastic differential equation (ISDE) representation for a system of infinite particles of jump type with long-range interaction. Our theorem can be applied to the systems of alpha-stable particles with logarithmic interactions associated with Dyson, Ginibre, Airy and Bessel random point fields, which are examined in the random matrix theory. In addition we also discuss the uniqueness of solutions of ISDEs. This is joint work with Hideki Tanemura (Chiba University).

On the fourth moment central limit theorem

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The so-called fourth moment phenomenon was first discovered by Nualart and Peccati (2005), who proved that a sequence of standardized multiple Wiener-Itô integrals \( \{F_n\} \) of fixed order converges in distribution to \( N(0,1) \) if \( E[F_n^4] \to 3 \). Combining Stein’s method with Malliavin calculus, Nourdin and Peccati (2009) obtained an elegant bound on the rate of convergence. We use zero-biased coupling to study the fourth moment phenomenon. This approach sheds light on how the fourth moment bound comes about in general situations. We apply the approach to obtain fourth moment bounds for independent sums, a combinatorial central limit theorem, and certain size-biased coupling. Extensions to non-normal approximations are discussed.

References


Persistence of Gaussian Stationary Processes: a spectral perspective

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Let \( f \) be a Gaussian stationary process (GSP) over \( \mathbb{R} \) or \( \mathbb{Z} \). What are possible asymptotic behaviors of its persistence probability
\[
P_f(N) := \mathbb{P}\left( f(t) > 0, \forall t \in (0, N) \right),
\]
as $N$ grows to infinity? What features of the covariance function determine this behavior?

This question, stated by Slepian in his seminal paper [5], has received much attention for over 50 years with old and new motivations stemming from probability, engineering and mathematical physics. Nonetheless, until recently, good estimates were known only for particular cases, or when the covariance kernel of the process is either non-negative or summable (see Dembo-Mukherjee [1]). A recent work by Krishna-Krishnapur [4] gave a general lower bound of $e^{-cN^2}$ on the persistence of any GSP over $\mathbb{Z}$ (under very mild conditions). This gave rise to other interesting questions: Is there a GSP that achieves persistence of the order of $e^{cN^2}$? Is it possible for a GSP over $\mathbb{R}$ to have even lower persistence?"

In the talk we discuss a new spectral point of view on persistence which greatly simplifies its analysis, and enables us to give nearly complete answers to the above questions (see [2, 3]). We will explain the methods leading to the following result.

**Theorem.** Let $f$ be a Gaussian stationary process over $\mathbb{R}$ or $\mathbb{Z}$. Suppose that its spectral measure is absolutely continuous with density $w(\lambda)$ satisfying $\int |\lambda|^\delta w(\lambda) d\lambda < \infty$ for some $\delta > 0$, and $e^{C\lambda^\alpha} \leq w(\lambda) \leq C\lambda^\alpha$ for all $\lambda$ in a neighborhood of 0 (and some $\alpha > -1$, $C, c > 0$). Then:

$$\log P_f(N) \begin{cases} 
\asymp -N^{1+\alpha} \log N, & \alpha < 0 \\
\asymp -N, & \alpha = 0 \\
\preceq -\alpha N \log N, & \alpha > 0.
\end{cases}$$

Moreover, if $w(\lambda)$ vanishes on an interval containing 0, then $\log P_f(N) \preceq -N^2$. In continuous time, it is possible to create examples for which $\log P_f(N) \preceq -e^{CN}$.\

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**References**


**Weighted dependency graphs, ASEP and the Ising model**

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The theory of dependency graphs is a powerful toolbox to prove asymptotic normality of sums of random variables, most of which are independent. We introduce an extension of this theory, called weighted dependency graphs, which enables to consider (weakly) dependent random variables as well.
This tool can be applied to the steady state of the symmetric simple exclusion process (and conjecturally to the asymmetric one), to the Ising model (the latter is joint work with Jehanne Dousse) and potentially to determinental point processes (work in progress).

References


Some comparison theorems for stochastic partial differential equations

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We discuss a few comparison principles for a class of stochastic heat equations and show how these results can be used to obtain precise information about the growth of the moments of the solutions. These comparison principles are a consequence of an approximation theorem which in turn relies on a local limit theorem which is of independent interest.

Continuous-state branching processes, extremal processes and super-individuals

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Consider a branching population model given by a flow of continuous-state branching processes (as Bertoin and Le Gall 2000). We characterize its long-term behaviour through subordinators and extremal processes. The extremal processes arise in the case of supercritical processes with infinite mean and of subcritical processes with infinite variation. The jumps of these extremal processes are interpreted as specific initial individuals whose progenies overwhelm the population. These individuals, which correspond to the records of a certain Poisson point process embedded in the flow, are called super-individuals. They radically increase the growth rate to $\infty$ in the supercritical case, and slow down the rate of extinction in the subcritical one. This notion of super-individuals allows us in particular to recover the so-called Eve property (defined in Duquesne and Labb 2014). This is based on a joint work with Chunhua Ma (Nankai university).
Exponential number of equilibria and depinning threshold for a directed polymer in a random potential

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Using the Kac-Rice approach, we show that the mean number $\langle N_{\text{tot}} \rangle$ of all possible equilibria of an elastic line (directed polymer), confined in a harmonic well and submitted to a quenched random Gaussian potential, grows exponentially $\langle N_{\text{tot}} \rangle \sim \exp r L$ with its length $L$. The growth rate $r$ is found to be directly related to the generalised Lyapunov exponent (GLE) which is a moment-generating function characterising the large-deviation type fluctuations of the solution to the initial value problem associated with the random Schrödinger operator of the 1D Anderson localization problem. For strong confinement, the rate $r$ is small and given by a non-perturbative (instanton, Lifshitz tail-like) contribution to GLE. For weak confinement, the rate $r$ is found to be proportional to the inverse Larkin length of the pinning theory. As an application, identifying the depinning with a landscape “topology trivialization” phenomenon, we obtain an upper bound for the depinning threshold $f_c$, in the presence of an applied force. The presentation is based on the preprint [1].

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References


Large deviations for the current of interacting particle systems

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I will illustrate the structure of the large deviations rate functionals for the current of interacting particle systems. I will consider the current flowing across a stationary non equilibrium state in a long time window and I will discuss the following issues. I will compare the results in the hydrodynamic scaling limit with a direct microscopic computation on the configuration space; I will discuss some exact results and finally I will discuss the existence of dynamic phase transitions.
Robust stopping times for detecting changes in distribution

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Let \(X_1, X_2, \ldots\) be independent random variables that are observed sequentially, \(X_1, \ldots, X_m\) have a common distribution function \(F_0\), while \(X_m, X_{m+1}, \ldots\) are all distributed according to \(F_1 \neq F_0\). It is assumed that two distributions are known, but the time change \(m\) is unknown and the goal is to estimate \(m\), or in other words, to find a stopping time \(M\) that detects the change-point \(m\) as soon as possible. The optimality of a stopping time \(M\) is understood in the following heuristic sense. For a given stopping time \(M\) we define the false alarm probability \(\alpha(M, m) = P_m\{M < m\}\) and we are looking for \(M^\circ\) minimizing

\[
E_m(M^\circ - m)_+ \quad \text{subject to} \quad \alpha(M^\circ, m) \leq \alpha
\]

for all \(m \geq 1\) and given \(\alpha \in (0, 1)\).

A standard mathematical approach to solving this problem is Bayesian. In this approach it is assumed that \(m\) is a random variable which doesn’t depend on \(X_1, X_2, \ldots\) and has a known distribution \(\pi(k) = P\{m = k\}\). In this case one can construct a stopping time \(M^\pi\) minimizing

\[
\sum_{m=1}^{\infty} \pi(m)E_m(M^\pi - m)_+ \quad \text{subject to} \quad \sum_{m=1}^{\infty} \pi(m)P_m\{M^\pi < m\} \leq \alpha
\]

(see, e.g. [1]). It is well known that \(M^\pi\) depends strongly on \(\pi(\cdot)\) and since the prior distribution is hardly known in practice, often Page’s CUSUM method [2] is used. It is also well known that CUSUM method has the false alarm probability 1.

The main goal in this talk is to propose a robust method of change-point detection which can be viewed as a compromise between Bayesian approach and CUSUM method. It is shown that the proposed method has a nearly minimal average delay of detection for given false alarm probability.

References


X-linked recessive disorders modelled through a multitype two-sex branching process

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Some defective alleles of certain genes can cause severe diseases or serious disorders in the organisms that carry them. Some of these genes, as could be those responsible for hemophilia, red-green color blindness or the Duchenne and Becker’s muscular dystrophies, are linked to X chromosome. If the alleles causing the disorder are dominant, all the carriers are affected and most of them do not reach breeding age so they are rarely detected in a population. However, recessive pernicious alleles -as the ones responsible of the diseases previously mentioned- can survive since
they only affect to carrier males and homozygous carrier females (the last ones must be daughters of a carrier male, so they rarely exist). Hence, heterozygous carrier females are not affected but can pass the allele onto offspring. Recently, in Gonzalez et al. (2016), a multitype two-sex branching process has been introduced to describe the evolution of the number of individuals carrying the alleles, \( R \) and \( r \), of a gene linked to X chromosome. The \( R \) allele is considered dominant and the \( r \) allele is supposed to be recessive and defective, responsible for a disorder. In the framework of this model we study conditions for the coexistence of both alleles in the population. The results are illustrated through a simulation-based study. Acknowledgements: This research has been supported by the Ministerio de Economía y Competitividad (MTM2015-70522-P), the Junta de Extremadura (GR15105) and the FEDER. References: Gonzalez, M., Gutierrez, C., Martinez, R., and Mota, M. (2016) Extinction probability of some recessive alleles of X-linked genes in the context of two-sex branching processes. In Branching Processes and their Applications, Lecture Notes in Statistics (del Puerto, I.M. et al., Eds), vol. 219, chapter 17. Springer-Verlag. DOI: 10.1007/978-3-319-31641-3

Branching processes with interactions, and their relation to population genetics

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In this talk we will introduce a generalisation of the Wright Fisher model, for a population with finite size and non-overlapping generations, allowing for several types of selection as well as simultaneous multiple mergers. The construction provides an almost sure dual relation between its frequency process and its ancestral process, and a bridge between population genetics and Branching processes with interactions. Finally, we study the long time behaviour of a wide family of branching processes with interactions that have a moment dual.

References


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Universal formula for the mean first passage time in planar domains

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We present a general exact formula for the mean first passage time (MFPT) from a fixed point inside a planar simply connected domain to a connected escape region on the boundary [1]. The underlying mixed Dirichlet-Neumann boundary value problem is conformally mapped onto the unit disk, solved exactly, and mapped back. The resulting formula for the MFPT is valid for an arbitrary space-dependent diffusion coefficient, while the leading logarithmic term is explicit, simple, and remarkably universal. In contrast to earlier works [2], we show that the natural small parameter of the problem is the harmonic measure of the escape region, not its perimeter. The conventional scaling of the MFPT with the area of the domain is altered when diffusing particles are released near the escape region. These findings change the current view of escape problems and related chemical or biochemical kinetics in complex, multiscale, porous or fractal domains, while the fundamental relation to the harmonic measure opens new ways of computing and interpreting MFPTs.

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Reflected BSDEs and non-linear optimal stopping: the irregular case

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Backward stochastic differential equations (BSDEs) have found number of applications in finance, among which pricing and hedging of European options, recursive utilities, risk measures. Reflected backward stochastic differential equations (RBSDEs) can be seen as a variant of BSDEs in which the (first component of the) solution is constrained to remain greater than or equal to a given process called the obstacle. Compared to the case of (non-reflected) BSDEs, there is an additional nondecreasing predictable process which keeps the (first component of the) solution above the obstacle. RBSDEs have been introduced by El Karoui et al. (1997) in the case of a continuous obstacle and have proved useful, for instance, in the study of American options. There have been several extensions of this work to the case of a discontinuous obstacle in all of which an assumption of right-continuity on the obstacle is made.

In this talk we present a further extension of the theory of RBSDEs to the case where the obstacle does not satisfy
any regularity assumption. Compared to the right-continuous case, the additional nondecreasing process, which "pushes" the (first component of the) solution to stay above the obstacle, is no longer right-continuous. We establish existence and uniqueness of the solution in appropriate Banach spaces. We characterize the solution in terms of the "value process" of an optimal stopping problem with non-linear \( g \)-expectation (where \( g \) is the driver of the RBSDE).

If time permits, we will also present the case of Doubly Reflected BSDEs whose barriers are not right-continuous and links with \( g \)-Dynkin games.

Our results use some tools from the general theory of processes (Mertens decomposition, Gal’chouk-Lenglart’s formula, ...) and some tools from the "classical" optimal stopping theory (with linear expectations).

References


An extreme class of nonnegative submartingales

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Let \( \mu \) be a probability measure on \( \mathbb{R}_+ \) with finite mean. Denote by \( Q(u), u \in (0,1) \), the lower quantile function of \( \mu \): \( Q(u) := \inf \{ x : \mu([0,x]) \geq u \} \). Define

\[
Q^*(u) := \int_0^u \frac{Q(t)}{1-t} \, dt.
\]

Then \( Q^*(u), u \in (0,1) \), is also the lower quantile function of a probability measure on \( \mathbb{R}_+ \) with the same mean as \( \mu \). We denote this measure by \( \mu^* \).

Consider now a nonnegative submartingale \( X = (X_t)_{t \geq 0} \) with \( X_0 = 0 \) on some filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})\). It is assumed that \( X \) is of class \((D)\), that is \( X \) admits the Doob–Meyer decomposition \( X = M + A \), \( M_0 = A_0 = 0 \), where \( M = (M_t)_{t \geq 0} \) is a uniformly integrable martingale and \( A = (A_t)_{t \geq 0} \) is a predictable increasing process, \( \mathbb{E}A_\infty < \infty \).

We assert that

- If \( \text{Law}(X_\infty) = \mu \), then
  \[
  \mathbb{E}f(A_\infty) \leq \int_0^1 f(Q^*(u)) \, du \quad \text{for any convex function } f.
  \]

- Given \( \mu \), there exists a filtered probability space and a submartingale \( X \) with \( \text{Law}(X_\infty) = \mu \) and satisfying the above assumptions such that we have equality in the above inequality for any convex \( f \).

- If \( X \) satisfies the above assumptions, \( \text{Law}(X_\infty) = \mu \), and equality holds in the above inequality for any convex \( f \), then
  \[
  \text{Law}(X_\infty, A_\infty) = \text{Law}(Q(U), Q^*(U)),
  \]
  where \( U \) is a random variable with the uniform distribution on \((0,1)\).
Moreover, we provide a full description of all submartingales $X$ from the previous item. In particular, a necessary condition on $X$ is that it belongs to the class $\langle \Sigma \rangle$ [1].

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References


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**Quickest detection with post-change drift uncertainty**

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We consider the Wiener disorder problem with unknown post change drift. Our approach is a mixture of min max with Bayesian. We propose a set of composite CUSUM rules that incorporate a useful statistic known as the CUSUM reaction period. We are able prove asymptotic optimality of the third order as the mean time to the first false alarm increases without bound.

**Acknowledgement:** This is joint work with H. Yang and M. Ludkovski.

References


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**Index Options and Volatility Derivatives via A Gaussian Random Field Risk-Neutral Density Model**

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**Boyu Wei** *The University of Hong Kong, Hong Kong*  
**Hailiang yang** *The University of Hong Kong, Hong Kong*

We propose a risk-neutral forward density model using Gaussian random fields to capture different aspects of market information from European options and volatility derivatives of a market index. The well-structured model is built in the framework of HeathJarrowMorton philosophy and Musiela parametrization with a user-friendly arbitrage-free condition. It implies the popular Geometric Brownian Model for the spot price of the market index and can be intuitively visualized to have a better view of the market trend. In addition, we develop theorems to show how our model drives local volatility and variance swap rates. Hence volatility futures and options can be priced taking the forward density implied by European options as the initialization input. And our model can be accordingly calibrated to the market prices of these volatility derivatives. An efficient algorithm is also developed for both simulating and pricing. And a comparative study is conducted between our model and existing models.
The Structure of Extreme Level Sets in Branching Brownian Motion

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Branching Brownian motion (BBM) is a classical process in probability, describing a population of particles performing independent Brownian motion and branching according to a Galton Watson process. Arguin et al. and Aïdékon et al. proved the convergence of the extremal process. In the talk we discuss how one can obtain finer results on the extremal level sets by using a random walk-like representation of the extremal particles. We establish among others the asymptotic density of extremal particles at a given distance from the maximum and the upper tail probabilities for the distance between the maximum and the second maximum (joint work with Aser Cortines and Oren Louidor).

Displacement exponent for loop-erased random walk on the Sierpinski gasket

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We consider a loop-erased random walk, which is constructed by erasing loops from a simple random walk, on the pre-Sierpinski gasket. We construct a consistent family of loop-erased random walks on the finite pre-Sierpinski gaskets, using the ‘erasing-larger-loops-first’ method, and extend it to a walk on the infinite pre-Sierpinski gasket. We also establish the asymptotic behavior of the walk as the number of steps increases, in particular, the displacement exponent and the law of the iterated logarithm.

**Theorem.** Loop-erased random walks on the finite pre-Sierpinski gaskets can be extended to a loop-erased random walk on the infinite pre-Sierpinski gasket.

**Theorem.** Let \( \lambda = \frac{20 + \sqrt{205}}{15} \) and \( \nu = \frac{\log 2}{\log \lambda} \). For any \( s > 0 \), there exist positive constants \( C_1(s) \) and \( C_2(s) \) such that

\[
C_1(s)n^{\nu s} \leq \mathbb{E} [|X(n)|^s] \leq C_2(s)n^{\nu s},
\]

where \( X(n) \) denotes the location of the loop-erased random walk starting at the origin after \( n \) steps and \( | \cdot | \) the Euclidean distance.

**Theorem.** There are positive constants \( C_3 \) and \( C_4 \) such that

\[
C_3 \leq \limsup_{n \to \infty} \frac{|X(n)|}{\psi(n)} \leq C_4, \text{ a.s.,}
\]

where \( \psi(n) = n^{\nu/2}(\log \log n)^{1-\nu} \).

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References

Parameter estimation for continuous-time branching processes observed at discrete times

Sophie HAUTPHENNE, The University of Melbourne, Australia, and EPFL, Switzerland

We develop a method based on saddlepoint approximation to estimate the parameters of a linear birth-and-death process whose population size is observed at discrete times. We then extend this method to multitype Markovian branching processes, separating the case where the individuals’ type can be observed from that where types are not observable.

Acknowledgement: Grant DE150101044

The markovian binary tree applied to the conservation of an endangered bird species

Sophie HAUTPHENNE, The University of Melbourne, Australia, and EPFL, Switzerland
Melanie Massaro, Charles Sturt University, Australia
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The Chatham Island black robin is an endangered songbird species that went nearly extinct in 1980. The population was able to recover thanks to intensive conservation efforts, and it reached about 250 adults in 2014. In this talk we model the black robin population using a continuous-time branching process called Markovian binary tree, which allows us to make an age-specific demographic analysis of the population. We estimate the model parameters based on a unique long-term dataset: we apply a non-linear regression method or a maximum likelihood method, depending on the precision of the available data. We discuss how the model outputs may be used within the field of conservation biology to inform future management of endangered island species.

Acknowledgement: Grant DE150101044

Parametric and Semiparametric Estimation Methods for Survival Data under a Flexible Class of Models

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In survival analysis, accelerated failure time (AFT) models are very useful in modeling the relationship between failure times and associated covariates, where covariate effects are assumed to appear in a linear form in the model. Such an assumption of covariate effects is, however, quite restrictive for many practical problems. To incorporate nonlinear relationship between covariates and transformed failure times, we propose partially linear single index models to facilitate complex relationship between failure times and covariates. We develop two inference methods which handle the unknown nonlinear function in the model from different perspectives. The first approach is weakly parametric which approximates the nonlinear function globally, whereas the second method is a semiparametric quasi-likelihood approach which focuses on picking up local features. To evaluate the validity of the proposed methods, we carefully explore the robustness issues to model misspecification, and establish the asymptotic properties for
the proposed methods. A real example is used to illustrate the usage of the proposed methods, and simulation studies are conducted to assess the performance of the proposed method for a broad variety of situations, including misspecification of the error distribution, or of the relationship between the response and covariates.

Random walks in cooling random environments
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We consider a model of a one-dimensional random walk in dynamic random environment that interpolates between two classical settings: (I) the random environment is resampled at every unit of time; (II) the random environment is sampled at time zero only. In our model the random environment is resampled along an increasing sequence of deterministic times. We consider the annealed version of the model, and look at three growth regimes for the resampling times: linear, polynomial and exponential. We prove laws of large numbers and central limit theorems. We list open problems and conjecture the presence of a crossover for the scaling behaviour in the polynomial growth regime.

Acknowledgement: Joint work with Luca Avena, Yuki Chino and Conrado da Costa.

Fluctuation properties of the stationary $q$-totally asymmetric simple exclusion process
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Tomohiro Sasamoto Tokyo Institute of Technology, Japan

The $q$-totally asymmetric simple exclusion process ($q$-TASEP), introduced in [1], is a version of the asymmetric exclusion processes: particles occupy sites of $\mathbb{Z}$ and each particle jumps to the right neighboring site with exclusion interaction, i.e. when the site is occupied by another particle, it cannot jump. While the jump rate is unity in the normal TASEP, it is generalized to $1/q$ gap in the $q$-TASEP, where $q \in [0, 1)$ is a parameter and gap means the number of empty sites between the particle and the right neighboring one.

The $q$-TASEP has been playing an important role in the recent progress in the integrable probability. Especially in a particular initial condition called the step initial condition, a Fredholm determinant representation for the $q$-Laplace transform of the pdf of the $N$th particle position has been obtained by using Macdonald difference operator [1] and duality technique [2]. However it has been known that these approaches cannot be applied to more generalized random initial condition such as the half stationary initial condition, due to the divergence of the $q$-moments.

In this talk, we will report our recent result on the $q$-TASEP with the half stationary initial condition [3]. We develop an alternative approach, which focus on the pdf of the particle position itself rather than its $q$-moments. Utilizing the Ramanujan summation formula and an elliptic version of the Cauchy determinant identity, we obtain a Fredholm determinant representation for the $q$-Laplace transform. From this representation, we can get the distribution function of the particle position for the stationary $q$-TASEP. In the long time limit, it is described by the Baik-Rains $F_0$ distribution. A few other limiting behaviors such as the scaling limit to the stationary OConnell-Yor polymer and the KPZ equation [4] will also be discussed.

References

Diffusion processes with weak constraint

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In this talk, we will present some results concerning the construction of some diffusion processes \((X_t; 0 \leq t \leq T)\), \(0 < T < \infty\), whose distribution is constrained to remain in some given space. Such processes appear in various theoretical and applied situations and, after a short presentation of some examples existing in the literature, we will focus on the approximation by means of a penalization method of such processes in the case where the constraint is of the form \(\mathcal{L}(X_t) \in K, \forall 0 \leq t \leq T\). The approximation, introduced in [3], is defined by introducing a family of diffusion processes \((X^\epsilon_t; 0 \leq t \leq T)\) where the Wasserstein distance \(W_2\) between the time-marginal distributions \(\mathcal{L}(X^\epsilon_t)\) and \(K\) is penalized by an order \(\epsilon > 0\). The resulting approximation \((X^\epsilon_t; 0 \leq t \leq T)\) is characterized as the solution of a singular stochastic differential equation whose well-posedness relies on some convex properties of the distance \(W_2\) and the differential calculus of functionals defined on the Wasserstein space \((\mathcal{P}_2, W_2)\), previously exhibited in [1], [2] and [4]. Under some technical assumptions on the constraint space \(K\), we show that the penalized approximation converges, as \(\epsilon\) tends to 0, to a diffusion process satisfying the given weak constraint.

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References

How Did the Population Start? Molecular Replication and Cell Reproduction Examples

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Consider a population which can only be observed when it has attained the size of a fraction of the habitat carrying capacity $K$, or some similar system size describing or determining entity, like the Michaelis-Menten constant in amplification of DNA (PCR). Other examples might be tumours or infections.

They start from a little number of individuals (DNA templates, mutants, invaders etc.) which is not observable. Can we find it later, when the population is observable? The astonishing answer is yes, in the ideal case when reproduction starts non-randomly, no otherwise. The difficulty lies in growth having a two-stage character: first the population is so little that individuals replicate freely, like in a branching process, but when the population can ultimately be observed, it interacts with its environment. $K$ is usually very large and as $K \to \infty$, the branching randomness concentrates to what looks like an initial veil of randomness, when size and time scale are normed by $K$.

Possibly this hints at a very general phenomenon of early randomness of free, expanding systems being interpreted as stochasticity of start only and the random initial condition of a system that is deterministic by law-of-large-numbers type effects.

Non-equilibrium Fluctuations of Interacting Particle Systems

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Using a reaction-diffusion model as a test bed, we show that the entropy production rate with respect to local equilibrium reference measures is of order $O(n^{d-2})$, improving on the bound $o(n^d)$ given by Yau's relative entropy method. This bound allows to prove a central limit theorem around its hydrodynamic limit for the density of particles in dimensions $d \leq 3$. The proof does not require explicit knowledge of the invariant measures of the systems, and therefore it is suitable to tackle various problems of interest of non-equilibrium statistical mechanics.

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Symmetry in financial models

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In this talk-considering the Black-Scholes framework- we describe the relationship between the financial models of prices of contingent claims and the geometric concept of symmetry. To this end we associate a manifold to the given market, construct a principal fiber bundle over this manifold the fibers of which are generated by the possible prices of the contingent claims for which the assets are underlying and numeraire transformations are transition functions. Choosing a section of this fiber bundle is equivalent to choosing a price for the claim at the corresponding time. The prices of the claims corresponding to a fiber are subject to the change of numeraire and change of time. We describe the structure group of the bundle and discuss the invariance of the equations describing the prices of the claims under the action of this group.

Keywords: Arbitrage, Contingent claim, Fiber bundle, Parallel translation, Symmetry
Classification: MSC 91B70, 91B24, 91B25

On numerical solutions of stochastic differential equations and deep learning-based approximation methods

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In this work we propose a new deep-learning based approximation algorithm for solving stochastic differential equations (SDEs). Numerical simulations in PYTHON using TENSORFLOW illustrate the efficiency of the proposed approximation algorithm. The talk is based on joint works with Weinan E (Princeton University, USA & Beijing University, China) and Jiequn Han (Princeton University, USA).

Generic cutoff at the entropic time for sparse exchangeable Markov chains

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We study convergence to equilibrium for a large class of Markov chains in random environment. The chains are sparse in the sense that in every row of the transition matrix P the mass is essentially concentrated on few entries. Moreover, the entries are exchangeable within each row. This includes various models of random walks on sparse random directed graphs. These models are generally non reversible and the equilibrium distribution is itself unknown. In this general setting we establish the cutoff phenomenon for the total variation distance to equilibrium, with mixing time given by the logarithm of the number of states times the inverse of the average row entropy of P. Joint work with Charles Bordenave and Pietro Caputo.
The ruin problem for Lévy-driven linear stochastic equations with applications to actuarial models with investments

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We deal with the asymptotic of the ruin probability for a process described by the linear SDE $dX^u = dP + X^u dR$, $X^u_t = u$, defined by a pair of independent Lévy processes $P$ and $R$. The setting is that of the model describing the evolution of the capital reserve of an insurance company selling the annuity, or a venture company selling innovations, invested in a risky asset with the price process $V$ upward jumps. We suppose that the cumulant-generating function $E \ln e^{-qV}$ of the increment of log price process $V = \ln \mathcal{E}(R)$ admits a root $\beta > 0$ at which $H$ is continuous while the business activity process $P$ has a negative drift and not too heavy tail of its Lévy measure. The main result implies a surprising corollary: if $R$ process $V$ has a negative drift and not too heavy tail of its Lévy measure, then the ruin probability always admits the exact asymptotic $Cu^{-\beta}$ as $u \to \infty$ without any further condition on $R$. Our approach is based on the Kesten–Goldie theorem on asymptotic of solutions of distributional equations combined with the recent result due to Guivarc’h and Le Page on positivity of the constant in this theorem. We provide also conditions under which the ruin happens with probability one.

Acknowledgement: Based on a joint work with S. Pergamenshchikov.

Rate of convergence for some class of stochastic networks with dynamic routing

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In this paper we consider a Jackson type queueing network with unreliable nodes. The network consists of $m < \infty$ nodes, each node is a queueing system of $M/G/1$ type. The input flow is assumed to be the Poisson process with parameter $\Lambda(t)$. The routing matrix $\{r_{ij}\}$ is given, $i, j = 0, 1, \ldots, m$, $\sum_{i=0}^{m} r_{ii} \leq 1$. The new request is sent to the node $i$ with the probability $r_{0i}$, where it is processed with the intensity rate $\mu_i(t, n_i(t))$. The intensity of service depends on both time $t$ and the number of requests at the node $n_i(t)$. Nodes in a network may break down and repair with some intensity rates, depending on the number of already broken nodes. Failures and repairs may occur isolated or in groups simultaneously. In this paper we assumed if the node $j$ is unavailable, the request from node $i$ is send to the first available node with minimal distance to $j$, i.e. the dynamic routing protocol is considered in the case of failure of some nodes. We formulate some results on the bounds of convergence rate for such case.

The following routing scheme for network nodes from the unreliable subset $D$ is assumed. Only transitions to $M_0 \setminus D$ are possible for nodes from $D$:

$$r^D_{ij} = \begin{cases} 0, & \text{if } j \in D, i \neq j, \\ r_{ij} + r_{ik}/s^D_{ik}, & \text{if } j \notin D, k \in D \\ \exists i \to j \to i' \to j' \to \ldots \to i'' \to k : s^1_{ij} * s^1_{ij'} * s^1_{ij''} * \ldots * s^1_{ik} \neq 0, & \end{cases}$$

where $p = \min\{2, 3, \ldots, m : s^p_{ik} \neq 0, \forall 1 < p \leq m\}$,

$$r_{ii} + \sum_{k \in D} s^p_{ik} r_{ik}, \text{ if } i \in M_0 \setminus D, i = j,$$

where $s^p_{ik}$ - element of a matrix $(s_{ij})^p$. 

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Definition. The markov process $X = (X(t), t \geq 0)$ is called unreliable queueing network if it’s defined by the following infinitesimal generator:

$$
\hat{Q}f(\tilde{n}) = \sum_{j=1}^{m} [f(T_{0j}\tilde{n}) - f(\tilde{n})]\Lambda(t)r_{0j}^D + \sum_{i=1}^{m} \sum_{j=1}^{m} [f(T_{ij}\tilde{n}) - f(\tilde{n})]\mu_i(n_i(t), z_i(t))r_{ij}^D + \sum_{i=1}^{m} [f(T_{ij0}\tilde{n}) - f(\tilde{n})]\mu_j(n_j(t), z_j(t))r_{0j}^D
$$

(2.2)

Theorem. If $X$ is a markov process with infinitesimal generator $Q$, it is assumed that $Q$ is bounded, the minimal intensity of service is strictly positive $\inf_{n_j, t} \mu_j(n_j, z_j) > 0$ and the routing matrix $(r_{ij}^D)$ is reversible, then $\text{Gap}(Q) > 0$, if the following condition is true: for any $i = 1, \ldots, m$, for the birth and death process, corresponding to the node $i$ with parameters $\lambda_i$ and $\mu_i(n_i, z_i)$ the spectral gap is strictly positive $\text{Gap}_i(Q_i) > 0$.

Diffusion approximations via Stein’s method and time changes

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We extend the ideas of (Barbour, 1990) and use Stein’s method to obtain a bound on the distance between a scaled time-changed random walk and a time-changed Brownian Motion. We then apply this result to bound the distance between a time-changed compensated scaled Poisson process and a time-changed Brownian Motion. This allows us to bound the distance between the Moran model with mutation and Wright-Fisher diffusion with mutation upon noting that the former may be expressed as a difference of two time-changed Poisson processes and the diffusion part of the latter may be expressed as a time-changed Brownian Motion. The method is applicable to a much wider class of examples satisfying the Stroock-Varadhan theory of diffusion approximation.

Multivariate functional approximations with Stein’s method of exchangeable pairs

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We combine the multivariate method of exchangeable pairs with Stein’s method for functional approximation and give a general linearity condition under which an abstract Gaussian approximation theorem for stochastic processes holds. We apply this approach to estimate the distance from a pre-limiting mixture process of a sum of random variables chosen from an array according to a random permutation and prove a functional combinatorial central limit theorem. We also consider a graph-valued process and bound the speed of convergence of the joint distribution of its rescaled edge and two-star counts to a two-dimensional continuous Gaussian process.

Acknowledgement: This is part of my PhD project supervised by Gesine Reinert.
Dynamical universality for infinite dimensional stochastic differential equations related to random matrices

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Universality of random matrices (or log-gasses), which is a central issue in random matrix theory, has been developed rapidly in the several decades. From these result, we obtain universal random point fields as the limit. Namely, finite particle systems in some class converge universal random point field with infinitely many particles as the number of particle to infinity. The limit objects are, for example, sine or Airy random point field, which are logarithmic correlated infinite particle systems. We can regard universality of random matrices as the central limit theorem for particle systems. Then, as a next issue, we would like to establish a dynamical counterpart of it, that is, we consider finite particle approximation for SDE related to random matrices. Give a finite particles dynamics as the finite dimensional SDE which is reversible with respect to above random point field with finite particles, and take a limit as particle number goes to infinity. In this case, because each particle interact with each other by logarithmic potential, which is long range potential, hence the limit transition is sensitive problem. Although it contains such a difficulty, we establish a general theorem for finite particle approximation. Under reasonable conditions, this result derives dynamical universality from universality of random matrices immediately. As examples, we can show universality for the Dyson Brownian motions, Airy processes, and so on. This talk is based on joint work with Hirofumi Osada in Kyushu university.

Multivariate fractional levy motion and its applications

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Since the beginning of the 90s many empirical studies of real telecommunication systems traffic have been conducted. It was found that traffic has some specific properties which are different from common voice traffic. More exactly it has the properties of self-similarity and long-range dependence and the distribution of loading size from one source has heavy tails. Some new models have been constructed where these features were captured. Brownian fractional motion and α-stable Lévy motion are well known examples. But both of these models have no all of above properties. Some more complicated models have been proposed using some combination of these ones. In particular we have proposed some univariate variant of fractional Levy motion ([1]).

In our report we consider multivariate analog of fractional Levy motion. This process is multivariate fractional brownian motion with random change of time where random change of time is Levy motion with one-sided stable distributions.

Definition. Multivariate fractional brownian motion (MFBM) with Hurst parameter $H$ $\in$ $(0,1)^d$ is a random process $Y = (Y_1(t), \ldots, Y_d(t), t \in \mathbb{R}^1)$ such that

1) $Y$ is a gaussian process;
2) $Y$ is self-similar with Hurst parameter $H$;
3) $Y$ has homogeneous increments.

We consider the case when $1/2 < H_p < 1$ for all $p: 1 \leq p \leq d$. Under additional restriction that the process $Y$ is invertable it has zero means and the following covariances:

$$
E(Y_p(s)Y_q(t)) = \frac{\sigma_{pq}}{2} \left( ||s||^{H_p+H_q} + ||t||^{H_p+H_q} - ||t-s||^{H_p+H_q} \right),
$$

(2.3)
where $\Sigma = (\sigma_{pq})$ is some positive definite matrix. (see [2]).

Let $(B_H(t), t \in \mathbb{R}^1)$ be a MFBM with Hurst parameter $H$ and covariance matrix $\Sigma$: $(L^1_H(t), t \geq 0), (L^2_H(t), t \geq 0)$ are standard $\alpha$-stable subordinators, $0 < \alpha < 1$; processes $B_H, L^1_H, L^2_H$ are independent.

**Definition.** Multivariate fractional Levy motion is a random process $X = (X(t), t \in \mathbb{R}^1)$ with values in $\mathbb{R}^d$ such that

$$X(t) := \begin{cases} B_H(L^1_H(t)), & t \geq 0, \\ B_H(-L^2_H(-t)), & t < 0. \end{cases}$$

We investigate the properties of this process and prove that it is self-similar and has stationary increments. Next we show that the coordinates of one-dimensional sections of this process have the distributions which are not stable. But asymptotic of tails for these distributions is the same as for stable ones. We apply this model to analyze of heterogeneous traffic and get some lower asymptotic bound of the probability of overflow at least one buffer (see the original ideas in [3]). There are other possible applications, for example in financial mathematics.

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**Stochastic simulation of advection-diffusion equation considering uncertainty in input variables**

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In environmental fluid mechanics, transport occurs through the combination of advection and diffusion. Due to the lack of understanding of the flow parameters, governing equations may be considered in the stochastic form. Karhunen–Loeve expansion (KLE) approach is applied to explore uncertainty based on the Advection-Diffusion equation (ADE). To assess the uncertainty, input variables are imposed in the framework of 1-D open channel flow. Our investigation is aimed at obtaining higher-order solutions to the statistical moments of the flow depth as random field. KLE approach is adopted to decompose the uncertain parameter in terms of infinite series containing a set of orthogonal Gaussian random variables. Eigenstructures of the covariance function associated with the input random parameter play a key role in computing the coefficients of the series and extracted from Fredholm’s equation. The flow depth is also represented as an infinite series which are obtained through decomposing by polynomial expansions. The coefficients of the last series are governed by a set of recursive equations that are derived from the ADE. Monte Carlo simulation (MCS), as a reliable approach, is carried out and compared with the KLE. It was found that
when higher-order approximations are used to represent initial condition, KLE results would be as accurate as MCS, however, with much less computational time and effort.

References


Stochastic recursions: between Kesten’s and Grey’s assumptions

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We study the stochastic recursions $R_n = A_n R_{n-1} + B_n$ and $R_n = \max\{A_n R_{n-1}, B_n\}$, where $(A_n, B_n) \in \mathbb{R} \times \mathbb{R}$ is an i.i.d sequence of random vectors and $R_0$ is an arbitrary initial distribution independent of $(A_n, B_n)_{n \geq 1}$. The tail behavior of their stationary solutions $R$ is well known under the so called Kesten-Grincevičius-Goldie or Grey-Grincevičius conditions. Under these assumptions, the tail of $R$ is roughly determined by $A$ or $B$ alone. It is natural to go a step further and to ask what happens when we are in a heavy tail regime, but neither Kesten’s nor Grey’s assumptions are satisfied. Is this “in-between” case much different from the known ones? The answer is affirmative.

In the most simplified version, our basic result says that if $A \geq 0$ a.s., $\mathbb{E} \log A < 0$ and there exists $\alpha > 0$ such that $\mathbb{E} A^\alpha = 1$, $\rho := \mathbb{E} A^\alpha \log A < \infty$, the law of $\log A | A > 0$ is non-arithmetic, $\mathbb{E} B^\alpha = \infty$ and $x \mapsto x^\alpha \mathbb{P}(B > x)$ is slowly varying, then

$$x^\alpha \mathbb{P}(R > x) \sim \frac{\mathbb{E} B^\alpha 1_{B \leq x}}{\alpha \rho},$$

where we write $f(x) \sim g(x)$ if $f(x)/g(x) \to 1$ as $x \to \infty$.

Under suitable conditions on $A$ and $B$ we find also the second order asymptotics of the tail of $R$. To obtain these results we prove renewal theorems that essentially generalize existing ones and are of independent interest, e.g.:

**Theorem.** Let $F$ denote a one-dimensional non-arithmetic probability distribution on $\mathbb{R}$. Define the renewal function by

$$H(x) = \sum_{n=0}^{\infty} F^{(n)}(x),$$

where $F^{(n)}$ denotes $n$-fold convolution of $F$ with itself. Assume that $F$ has first moment $\mu > 0$. For any slowly varying function $L$ such that $\int_0^x L(t)t^{-1}dt \to \infty$ as $x \to \infty$, one has

$$\int_{[0,x]} L(e^{x-z})dH(z) \sim \frac{1}{\mu} \int_0^{\exp x} \frac{L(t)}{t}dt.$$

This talk is based on joint work with Ewa Damek [1].

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References


Excited random walks in Markovian cookie environments

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Jonathon Peterson \textit{Purdue University, USA}

We consider a nearest-neighbor random walk on $\mathbb{Z}$ whose probability $\omega(x,n)$ to jump to the right from site $x$ depends not only on $x$ but also on the number of prior visits $n$ to $x$. The collection $\{\omega(x,n) : x \in \mathbb{Z}, n \in \mathbb{N} \cup \{0\}\}$, is sometimes called a “cookie environment” due to the following informal interpretation. Upon each visit to a site the walker eats a cookie from the bottom of a cookie stack at that site and chooses the transition probability according to the “flavor” of the cookie eaten. We assume that the cookie stacks are i.i.d. and that the cookie “flavors” at each stack $\omega(x,n)$, $n = 0, 1, \ldots$, follow a finite state Markov chain in $n$. Thus, the environment at each site is dynamic, but it evolves according to the local time of the walk at each site rather than the random walk time. We discuss recurrence vs. transience, ballisticity, and limit theorems for such walks.

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References


Random self-similar trees: dynamical pruning, invariance, and criticality

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We introduce generalized dynamical pruning on rooted binary trees with edge lengths. The pruning removes parts of a tree $T$, starting from the leaves, according to a pruning function defined on subtrees within $T$. The generalized pruning encompasses a number of previously studied discrete and continuous pruning operations, including the tree erasure and Horton pruning. For example, a finite critical binary Galton-Watson tree with exponential edge lengths is invariant with respect to the generalized dynamical pruning for arbitrary admissible pruning function. We will discuss other invariance results and applications.

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Stochastic monotonicity of Markov processes - A generator approach

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We consider stochastic orders on random variables which can be defined in terms of expectations of test functions. Notable examples are the standard stochastic order induced by the increasing functions or the convex order induced by convex functions, capturing the size and spread of random variables. In general, we consider cones of test functions characterized by $\Phi f \geq 0$ for some linear operator $\Phi$.

Of particular interest are stochastically monotone Markov processes which preserve stochastic order properties in time. The semigroup $\{S(t) : t \geq 0\}$ of a monotone Markov processes defined by $S(t)f(x) = E[f(X(t))|X(0) = x]$ maps these cones into themselves.

We introduce a new functional analytic technique based on the generator $A$ of the semi-group of a Markov process $X(t)$ and its resolvent to study the property of stochastic monotonicity. We show that the existence of an operator $B$ with positive resolvent such that $\Phi A - B\Phi$ is a positive operator for a large enough class of functions implies stochastic monotonicity. This establishes a technique for proving stochastic monotonicity and preservation of order for Markov processes that can be applied in a wide range of settings including various orders for diffusion processes with or without boundary conditions and orders for discrete interacting particle systems.

Joint work with Moritz Schauer (Leiden University, The Netherlands)

Limit Theorems for Functionals of Excursion Sets of Gaussian Random Fields

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There have been recent works on limit theorems of geometric functionals of random sets coming from discrete type models arising from various point processes (e.g. Blaszczyszyn et al.), or from models of smooth random fields (e.g. Adler & Naitzat, Estrade & León, Kratz & León, Marinucci & Cammarota, Marinucci & Vadlamani, Pham, Spodarev).

Reference

On stochastic population models with the Allee effect

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The paper presents the analysis of behavior of two types of the stochastic population model with the Allee effect, the one without delay, as well as the one with time dependent delay. Both models are represented by the stochastic differential equations. We prove the existence and uniqueness of positive solution of considered models. By the virtue of Lyapunov function and functional method, we find the sufficient conditions under which the population will become extinct, as well as the conditions for asymptotic mean square stability and stability in probability of the positive equilibrium states of the models. We are focused on the correlation between the growth rate \( r \) and population size \( N_t \), which is shown to be positive if the population size \( N_t \) is above the Allee threshold \( T \), and negative if \( N_t < T \). We also show that, for the delay model, if the initial population size exceeds environmental carrying capacity and time delay is sufficiently long, considered population is nonpersistent in mean. Finally, we put the theoretical results obtained through the paper into real life context. More precisely, we use reliable data for the populations of the gypsy moth \( Lymantria\\ dispersed \), African wild dog \( Lycaon\\ pictus \) and brown tree snake \( Boiga\\ irregularis \) and verify our theoretical findings.

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Adaptive complexity reduction in the spectral domain of music signals

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Acoustic signals exhibit a highly varying temporal structure. Therefore a change point detection procedure is needed, which captures the underlying signal structure. In the talk we present a novel change point detection criterion for music signals which relies on an explained variance criterion in the eigenspace of the constant-Q spectral domain. The change point detection procedure is used in combination with spectral complexity reduction method which mitigates effects of cochlear hearing loss. It is compared to a change point procedure based on equidistant boundaries. The numerical results show that the proposed change point detection criterion gives an improvement in terms of signal-to-artefacts ratio in comparison to corresponding equidistant boundaries procedure.

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References

Rigidity for the spectral gap on $\text{RCD}(K, \infty)$ spaces

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In this talk, we consider the rigidity problem for the spectral gap on $\text{RCD}(K, \infty)$ spaces ("Riemannian" metric measure spaces with "a lower Ricci curvature bound" by $K$). For weighted Riemannian manifolds, Cheng-Zhou [2] showed that the sharp spectral gap is achieved only when a one-dimensional Gaussian spaces is split off. In our framework, we employ the theory of regular Lagrangian recently developed by Ambrosio and Trevisan [1] to extend this result without relying on usual differentiable structure.

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References


Wiener–Hopf factorisation for Lévy processes with completely monotone jumps

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Fluctuation theory and Wiener–Hopf factorisation provides a way to study various properties of a Lévy process $X_t$. Most notably, it provides a description of the Laplace transform of the supremum functional $X_t$ in terms of either one-dimensional distributions of $X_t$ or its characteristic exponent $\Psi$. However, due to the presence of a bi-variate Laplace transform, this description is sometimes problematic. Our goal is to derive a more explicit expression for the supremum functional, as well as other objects appearing in fluctuation theory, by using complex-analytic methods.

When $X_t$ is a strictly stable Lévy process of index $\alpha$, then the density function of $X_t$ can be given as a Laplace-type integral involving a certain special function, the double sine function. More precisely, our representation is valid only when $\alpha > 1$ or $P(X_t > 0) \leq \frac{1}{2}$; otherwise the integral fails to converge in the usual sense, and special treatment is required. These results are a joint work with Alexey Kuznetsov from York University (Toronto), see [1].

The above result can be extended to more general Lévy processes $X_t$ that have completely monotone jumps: the Lévy measure of $X_t$ is assumed to have a density function $\nu$ such that $\nu(x)$ and $\nu(-x)$ are completely monotone functions on $(0, \infty)$. In addition, a number of (relatively mild) technical conditions is also imposed.

In this generality we can again express the density function of $X_t$ as a Laplace-type integral, involving expressions that depend on the analytic extension of $\Psi$. As it is the case for stable processes, the integral converges only when
either $\Psi$ grows sufficiently fast at infinity or upward jumps dominate (in an appropriate sense) downward jumps. Otherwise, again, the integral needs to be understood in an appropriate way.

The case of symmetric processes has been studied in [2,4]. The general case is still work in progress, preliminary results can be found in [3]. Our expressions can be used for numerical evaluation of the density of $\overline{X}_t$; however, due to their rather complicated nature, both efficiency and accuracy can be an issue here. On the other hand, our results seem to be well suited for theoretical study of the distribution of $\overline{X}_t$.

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References


A Dirichlet Process Characterization of Reflected Brownian Motion in a Wedge

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Reflected Brownian motion (RBM) in a wedge is a 2-dimensional stochastic process $Z$ whose state space in $\mathbb{R}^2$ is given in polar coordinates by $S = \{(r, \theta) : r \geq 0, 0 \leq \theta \leq \xi\}$ for some $0 < \xi < 2\pi$. Let $\alpha = (\theta_1 + \theta_2)/\xi$, where $-\pi/2 < \theta_2 < \pi/2$ are the directions of reflection of $Z$ off each of the two edges of the wedge as measured from the corresponding inward facing normal. We prove that in the case of $1 < \alpha < 2$, RBM in a wedge is a Dirichlet process. Specifically, its unique Doob-Meyer type decomposition is given by $Z = X + Y$, where $X$ is a two-dimensional Brownian motion and $Y$ is a continuous process of zero energy. Furthermore, we show that for $p > \alpha$, the strong $p$-variation of the sample paths of $Y$ is finite on compact intervals, and, for $0 < p \leq \alpha$, the strong $p$-variation of $Y$ is infinite on $[0,T]$ whenever $Z$ has been started from the origin. We also show that on excursion intervals of $Z$ away from the origin, $(Z,Y)$ satisfies the standard Skorokhod problem for $X$. However, on the entire time horizon $(Z,Y)$ does not satisfy the standard Skorokhod problem for $X$, but nevertheless we show that it satisfies the extended Skorkohod problem.
A statistical physics approach to Sine-beta processes

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We present a statistical physics approach for studying the Sine-beta process as minimiser of a certain free energy functional defined on the space of stationary point processes.

The flashing Brownian ratchet and Parrondo’s paradox

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A Brownian ratchet is a one-dimensional diffusion process that drifts toward a minimum of a periodic asymmetric sawtooth potential. A flashing Brownian ratchet is a process that alternates between a Brownian ratchet and a Brownian motion, producing directed motion. These processes have been studied by physicists and biologists for about 25 years. The flashing Brownian ratchet is the process that motivated Parrondo’s paradox, in which two fair games of chance, when alternated, produce a winning game. Parrondo’s games are relatively simple, being discrete in time and space. The flashing Brownian ratchet is more complicated. We show how one can study the latter process numerically using Markov chains.

References


Stochastic evolution equations and related optimal control problem

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We study parabolic stochastic partial differential equations in the framework of white noise analysis. Particularly, we consider a stochastic Cauchy problem of the form

$$\frac{\partial}{\partial t} U(t, x, \omega) = A U(t, x, \omega) + B \triangledown U(t, x, \omega) + F(t, x, \omega),$$

$$U(0, x, \omega) = U^0(x, \omega),$$

(2.4)

where \( t \in (0, T], \omega \in \Omega \) and \( u(t, \cdot, \omega) \) belongs to some Banach space \( X \). The operator \( A \) is densely defined, generating a \( C_0 \)-semigroup and \( B \) is a linear bounded operator which combined with the Wick product \( \triangledown \) introduces convolution-type perturbations into the equations. By applying the method of Wiener-Itô chaos expansions, also known as the propagator method, we reduce the SPDE to an infinite triangular system of deterministic PDEs, which can be solved by induction. Summing up all coefficients of the expansion and proving convergence in an appropriate weight space of stochastic processes, one obtains the solution of the initial SPDE.

In addition, we consider an optimal control problem related to a class of stochastic evolution equations (2.19) with quadratic cost functional. The optimal solution is thus obtained by combining techniques from white noise analysis with classical theory of optimal control.

References


Backward Stochastic Dynamics and Non-local Parabolic Partial Differential Equations and Extensions to Variational Inequalities

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It is well-known that backward stochastic differential equations (BSDEs for short) can provide probabilistic interpretation of solutions to corresponding partial differential equations in different senses. In this talk, we will build up a comprehensive analysis of the correspondence between backward stochastic dynamics (BSDs for short) in Brownian settings which can be regarded as a special class of BSDs on the general complete filtered probability space and weak solutions (instead of viscosity ones due to the intrinsic non-local nature of the integral of the gradient involved) of the following class of (non-local) parabolic partial differential equations:

$\begin{align*}
\frac{\partial u}{\partial t}(t,x) + Lu(t,x) + f(t,x,u(t,x),H(t,x,(\sigma^*\nabla u)(t,\cdot))) &= 0, \\
u(T,x) &= g(x).
\end{align*}$

And by extending the unique existence of the general BSDs to BSDs with a subdifferential operator, which is also called as backward stochastic dynamical variational inequalities (BSDVIs for short), we also establish the connection between these BSDVIs and the weak solutions of a class of (non-local) parabolic variational inequalities, which is barely touched in the existing literature due to its unconventional setting.

Remark. This talk is based on the paper [3], which has been submitted to the SPA and is now accepted with minor revision.

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References


Continuous-State Branching Processes in Lévy Environments

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A continuous-state branching processes in random environment (CBRE-process) is defined as the strong solution of a stochastic integral equation. The environment is determined by a Levy process with no jump less than -1. Characterizations of the quenched and annealed transition semigroups of the process can be given in terms of a backward stochastic integral equation. The process hits zero with strictly positive probability if and only if its branching mechanism satisfies Grey’s condition. In that case, a characterization of the extinction probability is given using a random differential equation with singular terminal condition. The strong Feller property of the CBRE-process can be established by a coupling method. A criterion for the ergodicity of a subcritical CBRE-process with immigration can be given by an integral condition. This is a brief survey of the recent progresses in the subject.

Martingale Optimal Transport in the Euclidean space

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We introduce the martingale optimal transport problem and its dual problem. While the existence of solutions to the primal problem is rather standard, it is not so for the dual problem and we discuss the issue. Then as an application of the dual attainment, we show some structural results for the martingale optimal transport plans in higher-dimensional Euclidean spaces.

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References


General linear-fractional branching processes with discrete time

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The linear-fractional Bienaymé-Galton-Watson processes is a well known case when many characteristics of a branching process can be computed explicitly. In [1] we study a linear-fractional Bienaymé-Galton-Watson process with a general type space. The corresponding tree contour process is described by an alternating random walk with the downward jumps having a geometric distribution. This leads to the linear-fractional distribution formula for an arbitrary observation time, which allows us to establish transparent limit theorems for the subcritical, critical and supercritical cases. Our results extend recent findings for the linear-fractional branching processes with countably many types.

References


Large deviations for branching processes in random environments

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Let \((Z_n)\) be a supercritical branching process in an independent and identically distributed random environment. We show Cramér’s large deviation expansion (also called moderate deviation expansion) for \((\log Z_n)\). In the proof we show a Berry-Esseen bound on the rate of convergence in the central limit theorem for \((\log Z_n)\), improve an earlier result about the harmonic moments of the limit variable of the naturally normalized population size, and use in an adapted way Cramér’s change of probability for the associated random walk. Some related results on the lower large deviation of \(Z_n\), on the harmonic moments of \(Z_n\), and on the asymptotic distribution of \(Z_n\), will also be presented briefly. (Mainly based on the paper whose reference is given below)

References

Absolute continuity under time shift of trajectories and related stochastic calculus

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The paper is concerned with a class of two-sided stochastic processes of the form $X = W + A$. Here $W$ is a two-sided Brownian motion with random initial data at time zero and $A = A(W)$ is a function of $W$. Elements of the related stochastic calculus are introduced. In particular, the calculus is adjusted to the case when $A$ is a jump process. Absolute continuity of $(X, P_\lambda)$ under time shift of trajectories is investigated. For example under various conditions on the initial density with respect to the Lebesgue measure, $m = d\nu/d\lambda$, and on $A$ with $A_0 = 0$ we verify

$$\frac{P_\lambda(dX_{-t})}{P_\lambda(dX)} = \frac{m(X_{-t})}{m(X_0)} \prod_i |\nabla W_i X_{-t}|$$

a.e. where the product is taken over all coordinates. Here $\sum_i (\nabla W_i X_{-t})_i$ is the divergence of $X_{-t}$ with respect to the initial position. Crucial for this is the temporal homogeneity in the sense that $X(W_{t+v} + A_0 I) = X(W_{t+v})$, $v \in \mathbb{R}$, where $A_0 I$ is the trajectory taking the constant value $A_0(W)$. In the talk we will present the major steps in order to prove such results.

By means of a such a density, partial integration relative to the generator of the process $X$ is established. A further application is relative compactness of sequences of processes of the form $X^n = W + A^n$, $n \in \mathbb{N}$.

References


Gradient flows and quantum entropy inequalities via matrix optimal transport

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We present a new class of transport metrics for density matrices, which can be viewed as non-commutative analogues of the 2-Wasserstein metric. With respect to these metrics, we show that dissipative quantum systems can be formulated as gradient flows for the von Neumann entropy under a detailed balance assumption. We also present geodesic convexity results for the von Neumann entropy in several interesting situations. These results rely on an intertwining approach for the semigroup combined with suitable matrix trace inequalities. This is joint work with Eric Carlen.

References

Cooperative branching on trees and other lattices

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Let \( \Lambda(V, E) \) be a countable, connected, vertex transitive, locally finite graph with a vertex set \( V \) and a set of (undirected) edges \( E \). We consider a Markov process \( X \) with values on \( \{0, 1\}^\Lambda \) and the following dynamics. At rate \( \alpha \) each two neighbouring occupied sites produce an offspring on an empty site adjacent to (at least) one of them (cooperative branching), at rate \( \mu \) occupied sites become empty (death) and at rate \( \gamma \) particles at occupied sites ”move” to one of the neighbouring sites (random walk dynamics). We are in particular interested in the phase transitions of this process on different lattices, namely we would like to estimate the critical branching rate \( \alpha_{\text{surv}} \) for the probability of survival and \( \alpha_{\text{app}} \) for the existence of a non-trivial upper invariant law of the process \( X \). In the talk, we will focus on the case when \( \gamma = 0 \) and on processes on regular trees and complete graphs as well as a process which is dual to a mean-field model with the aforementioned dynamics.

A new approach in classification discrete survival data by discrete hidden Markov model and comparison to one common method

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Background: Diagnosis of cancer survival is very important and existence of reliable system is necessary for reduce the medical expenditure. Discrete Hidden Markov Models (DHMMs) are a ubiquitous tools and probabilistic techniques for modeling sequence data. These models include an underlying stochastic process that is hidden but could be inferred through the observations it generates. Discrete time Survival data comprised of a set of events and observation during the discrete time. This paper presents the performance of DHMM in classification of event and compere results with logistic regression that considers overall probability of occurrence of event.

Methods: A dataset was acquired of Health Department and Cancer registry of Kerman Province which is located in south of Iran, and data includes the information of 900 breast cancer patients aged 15 to 80 years, and its seven associated risk factors among patients since 1999 to 2007. Our breast cancer data are assumed to include two possible events such as live and died. DHMM was trained based on a set of 675 patients information (75% of data) and that was validate in a test set of 225 patients information (25% of data). The Area Under the ROC Curve (AUC), sensitivity, specificity and accuracy used as measures of validate of the efficiency of model in prediction of patients survival status.

Results: Sensitivity, specificity, accuracy and the and area under the ROC curve of the logistic regression was 0.86, 0.97, 0.92 and 0.951, respectively. The sensitivity, specificity, accuracy and the area under the ROC curve of DHMM was 0.989, 0.99, 0.939 and 0.94, respectively.
Conclusions: According to the four evaluation criteria, DHMM would give better performance than logistic regression.

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References


Infinitely ramified measures and branching Lévy processes

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An infinitely ramified point measure is a point measure that can be written, for any \( n \in \mathbb{N} \) as the value at time \( n \) of a branching random walk. We prove that such a measure can be represented as the value at time 1 of a branching Lévy process: a particle process in which every individual produces offspring and moves according to some Poissonian dynamic. This result can be thought of as a branching process analogue of the correspondence between Lévy processes and infinitely divisible random variables. In the rest of the abstract, we give a more precise description of this result.

We denote by \( \mathcal{P} \) the set of point measures that put finite mass to \([0, +\infty)\). This set is identified with the set of non-increasing sequences \((x_j) \in (\mathbb{R} \cup \{-\infty\})^\mathbb{N}\) that converge to \(-\infty\) observing that

\[
\mu \in \mathcal{P} \iff \mu = \sum_{j \in \mathbb{N}} \delta_{x_j}, \text{ with } x_1 \geq x_2 \geq \cdots \text{ and } \lim_{n \to +\infty} x_j = -\infty.
\]

We introduce the shift operator \( \tau_x \mu = \sum_{j \in \mathbb{N}} \delta_{x_j+x} \) on \( \mathcal{P} \).

**Definition** (Infinitely ramified measure). Let \( \mu \) be a random variable taking values in \( \mathcal{P} \). A branching random walk is a process \((Z_n, n \in \mathbb{N})\) that can be defined as follows: given \((\mu_{n,j}, j, n \in \mathbb{N})\) i.i.d. random variables with same law as \( \mu \), we set

\[
Z_0 = \delta_0 \quad \text{and} \quad Z_{n+1} = \sum_{j=1}^{\mu(n+1), j} \tau_{x_j} Z_n,
\]

where \((z_{n,j}, j \in \mathbb{N})\) is the sequence of atoms associated to \( Z_n \).

An infinitely ramified measure is a random element \( Z \) of \( \mathcal{P} \) such that for any \( n \in \mathbb{N} \), there exists a branching random walk \( Z^{(n)} \) such that \( Z^{(n)} = Z \) in distribution.

The precise definition of branching Lévy processes being a little bit technical, we only give here a rough picture. Let \( \sigma^2 > 0 \), \( a \in \mathbb{R} \) and \( \Lambda \) a measure on \( \mathcal{P} \) such that for some \( \theta > 0 \),

\[
\int_{\mathcal{P}} \sum_{j \geq 1} e^{\theta x_j} - 1 - \theta x_j \mathbb{1}_{|x_j| \leq 1} \Lambda(d(x_j)) < +\infty.
\]
Let \( N \) be a Poisson point process with intensity \( dt \), we define \( N \text{jump} \) and \( N \text{repro} \) the image measures of \( N \) by the applications \( (t, (x_j)) \mapsto (t, x_1) \) and \( (t, (x_j)) \mapsto (t, (x_{j+1})) \) respectively. A branching Lévy process with characteristics \((\sigma^2, a, \Lambda)\) is a particle process on \( \mathbb{R} \) in which each individual moves according to a Brownian motion with variance \( \sigma^2 \), drift \( a \), and jump measure \( N \text{jump} \). For each atom of \((t, (y_j))\) of \( N \text{repro} \), the individual produces offspring at time \( t \) at distance \( y_1, y_2, \ldots \) from its current position.

**Theorem.** Let \( Z \) be an infinitely ramified process. If there exists \( \theta > 0 \) such that \( \mathbb{E} \left( \int e^{\theta z} dZ \right) \), then there exists a càdlàg extension of \( Z \) to a process on \( \mathbb{R}_+ \). Moreover, this càdlàg extension is a branching Lévy process with characteristics \((\sigma^2, a, \Lambda)\).

References


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**An interacting particles system arising from random solutions to the Burgers’ equation**

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I will introduce a stochastic particles system, which can be mapped to random solutions to the deterministic Burgers’ equation. There are two types of particles moving at deterministic speed, and subject to annihilation and random creation phenomena. Motivations, reversibility and long time behavior will be discussed.

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**Stability for some stochastic fractional systems**

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We will first show the stability for some linear stochastic fractional systems. Then, we will generalize this type the result giving the stability for a class of semi linear fractional stochastic integral equations. The more general case is considering an equation with a function as initial condition and with additive noise, which is a Young integral that could be a functional of fractional Brownian motion.

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**Empiristic probability theory: a new view of Von Mises collectives**

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Probability theory, as established by Kolmogoroff (1933), is usually taught without sufficient clarification of its link to the physical world. Such a clarification was in principle provided by von Mises through the introduction of
collectives in several publications from 1919 onwards, but in a somewhat informal and initially, under some natural
formal interpretation, even inconsistent way. Although a satisfactory formalization (of collectives as such, but not of
their role as links between the world and classical probability theory) and an existence theorem were soon obtained
by Wald (1937) and improved by Feller (1939), the von Mises approach (which we call empiristic probability theory)
apparently didn’t really catch on.

With the aim of clarifying empiristic probability theory a bit further, we propose in our “one definition, no
theorem” talk a formalization of the role of collectives as links between a (mathematical, more precisely Boolean)
world and probability theory. In the empiristic probability theory thus obtained, of course probabilities are limits of
relative frequencies, and for example the formula for elementary conditional probabilities is a theorem, but also, and
importantly we think, it becomes very obvious that world events don’t have probabilities.

References

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The infinite dimensional stochastic linear quadratic optimal control problem

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Optimal control of systems governed by stochastic partial differential equations arise naturally in science and
engineering. We consider stochastic optimal control problems where the state equation is linear and the cost functional
is quadratic. We show that the optimal control is given in feedback form in terms of a Riccati equation. We investigate
the numerical approximation of the problem, in particular, the convergence of Riccati operators and the numerical
solution of the state equation. Moreover, we proposed a new numerical scheme based polynomial chaos expansion
to approximate the optimal control directly. Numerical experiments of specific applications show the performance of
our approach.

References


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Statistical inference in Controlled Branching Processes via disparity measures

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Branching processes are appropriate probabilistic models for the description of population dynamics. In this work, we focus our attention on controlled branching processes, in which individuals reproduce independently of the others following the same probability distribution, known as offspring distribution, and moreover, the number of progenitors with reproductive capacity in each generation is controlled by a random control function. The aim of this talk is to provide estimators of the offspring parameters based on the sample given by the entire family tree up to a certain generation. Assuming a parametric context for the offspring distribution, we make use of disparity measures. We analyse the asymptotic and robust properties of the proposed estimators (see [1] and [2] for further details). The results are illustrated by a simulated example.

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References


Limit laws for fermionic fields on hierarchical lattice

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Let $p$ be a natural number, $p > 1$. The hierarchical lattice $T_p^d$ can be represented as the lattice $Z^d$ of integer-valued $d$-dimensional integer vectors endowed with hierarchical distance $d_p(i, j)$. Let $k = (k_1, \ldots, k_d) \in Z^d$, $s \in N$,

$$V_{k,s} = \{j \in Z^d : (k_l - 1) p^s < j_l \leq k_l p^s, \ l = 1, \ldots, d\}.$$ 

The hierarchical distance $d_p(i, j)$, $i, j \in Z^d$ is defined as $d_p(i, j) = p^{s(i,j)}$, if $i \neq j$, where $s(i,j) = \min\{s : \text{there is } k \text{ such that } i \in V_{k,s}, \ j \in V_{k,s}\}$. Four-component spins $\psi^*(i) = (\psi_1(i), \psi_2(i), \psi_3(i), \psi_4(i))$, whose components are the generators of the Grassmann algebra, are located at the nodes of the hierarchical lattice $T_p^d$. The hierarchical fermionic model is given by the Hamiltonian

$$H(\psi^*; \alpha) = H_0(\psi^*; \alpha) + \sum_{i \in T_p^d} L(\psi^*; r, g),$$
where the Gaussian part of the Hamiltonian
\[
H_0(\psi^*; \alpha) = \sum_{i,j \in T^d_p} d_0(i, j) [\bar{\psi}_1(i) \psi_1(j) + \bar{\psi}_2(i) \psi_2(j)],
\]
d_0(i, j) = a_1(\alpha) d_p^{-\alpha}(i, j), i \neq j, d_0(i, i) = a_2(\alpha), a_1 \text{ and } a_2 \text{ are normalizing constants.}

The non-Gaussian part of the Hamiltonian is given by the Lagrangian
\[
(L(\psi^*; r, g) = r \left( \bar{\psi}_1(i) \psi_1(i) + \bar{\psi}_2(i) \psi_2(i) \right) + g\bar{\psi}_1(i) \psi_1(i) \bar{\psi}_2(i) \psi_2(i),
\]
where \(r\) and \(g\) are real-valued coupling constants of the model and \(\alpha\) is a real-valued parameter of the model. The block-spin transformation of the Kadanoff-Wilson renormalization group is defined by the formula
\[
r(\alpha)\psi^*(i) = p^{-\alpha/2} \sum_{j \in V_i} \psi^*(j),
\]
where \(\alpha\) is the renormalization group parameter. We use the notion of the Grassman-valued "density" of the free measure \(f(\psi^*) = \exp\{-L(\psi^*; r, g)\}\) instead of the Lagrangian \(L(\psi^*; r, g)\). In the general case the "density" of the free measure is given by \(f(\psi^*; c) = c_0 + c_1(\bar{\psi}_1 \psi_1 + \bar{\psi}_2 \psi_2) + c_2 \bar{\psi}_1 \psi_1 \bar{\psi}_2 \psi_2\). A triple \(c = (c_0, c_1, c_2)\) is treated as a point in the two-dimensional real projective space \(RP^2\). The Gaussian part is invariant under this renormalization group (RG) transformation. The RG transformation does not change the structure of the non-Gaussian part and is reduced to the transformation \(R(\alpha)\) in the projective \(c\)-space: \(R(\alpha)(c_0, c_1, c_2) = (c'_0, c'_1, c'_2)\),

\[
\begin{align*}
c'_0 &= \left( (c_1 - c_0)^2 + \frac{1}{n} (c_0 c_2 - c_1^2) \right), \\
c'_1 &= \lambda \left( (c_1 - c_0)(c_2 - c_1) + \frac{1}{n} (c_0 c_2 - c_1^2) \right), \\
c'_2 &= \lambda^2 \left( (c_2 - c_1)^2 + \frac{1}{n} (c_0 c_2 - c_1^2) \right),
\end{align*}
\]
where \(\lambda = p^{\alpha-d}, n = p^d\) is the size of the elementary cell of the hierarchical lattice \(T^d_p\). The transformation \(R(\alpha)\) has trivial (Gaussian) fixed point \((1, 0, 0)\), and fixed point \((0, 0, 1)\), given by the Grassmann \(\delta\) function \(\delta(\psi^*) = \bar{\psi}_1 \psi_1 \bar{\psi}_2 \psi_2\).

We denote this fixed point by \(\delta\). For \(\alpha \neq d\) the RG map has also two non-Gaussian fixed points. If \(\alpha > d\), then the only attracting fixed point of RG transformation is \(\delta\). If \(\alpha > 3d/2\), then Gaussian point is unstable fixed point and both non-Gaussian fixed points are saddle points. We describe explicitly zone structure of attraction domain of the \(\delta\)-fixed point and show that global RG flow has a nice description in terms of this zone structure. We also describe global behavior of stable RG-invariant curves for non-Gaussian fixed points.

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**Parameter estimation of CARMA processes with periodically correlated increments**

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There are many studies on developing continuous-time processes suitable for analyzing the volatilities which are non-stationary. For finding a model that imitates the properties of a data and easy to predict the future of the data, it is necessary to discuss on estimating the parameters of the model. For this we estimate parameters of the
semi-Levy driven continuous-time autoregressive moving average (SLCARMA) process with the order \((p, q)\) which is periodically correlated. A discrete time process with \(q < p\) to regularly spaced data is fitted. Then we attempt to find the parameters whose values at the observation times have the same distribution as the values of the fitted process at those times. The estimation of the SLCARMA coefficients are obtained from the sample processes by least squares of each stationary elements of corresponding multidimensional stationary process. We implement the Kalman recursion technique to the parametric estimation of in conjunction with the state-space representation of associated processes. An empirical procedure for estimating the parameters of the SLCARMA model for a financial market are studied.

**Model**

By some conditions, the solution of the stochastic differential equation by it's state-space representation

\[ Y(t) = bX(t) \text{ and } dX(t) - AX(t)dt = edS(t) \]

converges to

\[ X(t) = \int_{-\infty}^{t} e^{A(t-u)} edS(u) \]

where \(S(t) = \gamma t + \sum_{k=1}^{N(t)} J_k\) is a semi-Levy process and \(\gamma \in \mathbb{R}\), \(\{N(t), t \geq 0\}\) is a simple semi-Levy Poisson process with parameter \(\Lambda_t\), defined by \(\Lambda_t = (k-1) \sum_{i=1}^{k-1} \lambda_i + \sum_{i=1}^{k-1} \lambda_i + \frac{\lambda_i J_i}{t} \) for \(t = (k-1)T + s\). Also \(\{J_k, k \geq 1\}\) is an independent identically distributed sequence of random variables with probability distribution \(F\). The first and second order of such \(S(t)\) are \(E[S(t)] = \gamma t + \Lambda_t \kappa\) and \(\text{var}[S(t)] = \Lambda_t \beta\) where \(E[J] = \kappa, E[J^2] = \beta\) and \(E[N(t)] = \text{var}[N(t)] = \Lambda_t\).

We find that the first order moment and the covariance function of the solution is periodically correlated with period \(T\). Therefore \(Y(t)\) is the SLCARMA process and we are to estimate the coefficients of the introduced process by estimation the coefficients of the corresponding multidimensional stationary CARMA processes.

**References**


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**The inviscid Burgers equation with fractional Brownian initial data: the dimension of regular Lagrangian points**

George MOLCHAN MITP, Russia

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Fractional Brownian motion, H-FBM, of index \(0 < H < 1\) is considered as initial velocity in the inviscid Burgers equation. It is shown that the Hausdorff dimension of regular Lagrangian points at any moment \(t\) is equal to \(H\). This fact validates the Sinai-Frisch conjecture known since 1992. We find also that the integrated H-FBM does not exceed a fixed positive level in the interval \((-T, T)\) with probability having the log-asymptotics: \((H - 1 + o(1))logT\).
Wong-Zakai Approximation for Stochastic Moving Boundary Problems

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Moving boundary problems allow for modeling of multi-phase systems with separating boundaries evolving in time. While the deterministic case is in general well understood, introducing stochastic terms makes the analysis more complicated. In this talk, we discuss a class of stochastic second-order PDEs in one space dimension with possibly non-linear, Stefan-type condition on the boundary interaction. Existence results have been recently proven in the context of stochastic evolution equations on interpolation spaces. Extending results from Nakayama, we prove a Wong-Zakai type approximation result for these local solutions. This can be used to extend results from deterministic theory to stochastic problems, e.g. forward invariance of closed sets and non-negativity of solutions.

References


Approximating the equilibrium quantity traded and welfare in large markets

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Konstantin Borovkov The University of Melbourne, Australia

We consider the efficient outcome of a canonical economic market model involving buyers and sellers with independent and identically distributed random valuations and costs, respectively. When the number of buyers and sellers is large, we show that the joint distribution of the equilibrium quantity traded and welfare is asymptotically normal. Moreover, we bound the approximation rate. The proof proceeds by constructing, on a common probability space, a representation consisting of two independent empirical quantile processes, which in large markets can be approximated by independent Brownian bridges. The distribution of interest can then be approximated by that of a functional of a Gaussian process. This methodology applies to a variety of mechanism design problems.

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Discretisation schemes for level sets of planar Gaussian fields

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Gaussian random fields are prevalent throughout mathematics and the sciences, for instance in physics (wavefunctions of high energy electrons), astronomy (cosmic microwave background radiation) and probability theory (connections to SLE, random tilings etc). Despite this, the geometry of such fields, for instance the connectivity properties of level sets, is poorly understood. In this talk I will discuss methods of extracting geometric information about levels sets of a planar Gaussian random field through discrete observations of the field. In particular, I will present recent work that studies three such discretisation schemes, each tailored to extract geometric information about the levels set to a different level of precision, along with some applications.

Properties of Persistence Exponents

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Given a discrete time real valued stochastic process \((X_i)_{i \geq 0}\), we say it has persistence exponent \(\theta \in [0,1]\) if \(\theta\) satisfies
\[
P(X_0 > 0, \ldots, X_n > 0) = n^{\theta + o(n)}.
\]
Persistence exponents have been a recurring theme of interest in Probability Theory, with most of the existing literature focusing on existence of the exponent. In this talk, we study both existence and properties of persistence exponents in two concrete examples, the Auto Regressive Process of order \(p\) and the Moving Average Process of order \(q\), as the parameters of the underlying process vary.

For an AR\((p)\) process with parameters \((a_1, \ldots, a_p)\), our main result is that the persistence exponent on the non negative orthant is strictly increasing on the domain \(\sum_{i=1}^{p} a_i < 1\) for log concave innovation distributions, and is a constant on the domain \(\sum_{i=1}^{p} a_i > 1\). We also show that the exponent is always positive, but can equal 1.

For an MA\((q)\) process with parameters \((b_1, \ldots, b_q)\), we show that the persistence exponent is zero on the domain \(\sum_{j=1}^{q} b_j = -1\), and is positive if \(\sum_{j=1}^{q} b_j \neq -1\). In this case, the exponent is always less than 1.

The main tool for the results is to set up an eigenvalue equation for which the persistence exponent is the leading eigenvalue. Using this approach, we are able to compute the exponents explicitly for some AR and MA processes involving uniform and exponential innovation distributions.

Branching random walks in random environment

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We consider the branching random walks in random environment(BRWRE) in which the offspring distributions are given as i.i.d.time-space random variables. It is known that the phase transition occurs: the normalized point
process associated with BRWRE satisfies the central limit theorem when the environment is not strong (weak disorder phase), whereas localizations are investigated when the environment is strong enough (strong disorder phase).

In this talk, we focus on the detail of the central limit theorem in the weak disorder.

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References


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Gaussian comparison and anti-concentration inequalities for norms of Gaussian random elements

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Vladimir Ulyanov Lomonosov Moscow State University, Russia

We derive a bound on the Kolmogorov distance between the probabilities of two Gaussian elements to hit a ball in a Hilbert space. The key property of these bounds is that they are dimensional-free and depend on the Schatten 1-norm of the difference between the covariance operators of the elements. We are also interested in the anti-concentration bound for the squared norm of a non-centered Gaussian element in a Hilbert space. All bounds are sharp and cannot be improved in general. We provide a list of motivation examples and applications for the derived results as well.

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Exceedances of high levels till a stopping time

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We deal with the distribution of the largest observation $M_t$ till a stopping time $\mu(t)$, and the distribution of the number $N_x(t)$ of those observations till a stopping time, which exceed a given level $x$. We establish sharp uniform estimates of the rate of convergence as well as asymptotic expansions in the limit theorems for $M_t$ and $N_x(t)$. As an application of our results we consider the problem of the length of the longest interval between consecutive jumps of a Poisson process.

References

On the behaviour of stochastic heat equations on bounded domains

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We consider the following stochastic heat equation

$$\partial_t u_t(x) = \frac{1}{2}\Delta u_t(x) + \lambda \sigma(u_t(x)) \tilde{W}(t, x)$$

on a bounded domain, where the noise is white in time and correlated in space. Under Dirichlet boundary conditions, we show that in the long run, the moments of the solution grow exponentially fast if $\lambda$ is large enough. But if $\lambda$ is small, then the moments eventually decay exponentially, which implies the non-intermittence of the solution for small $\lambda$. This talk is based on a joint work with Mohammud Foondun.

References


A large deviations analysis of parallel tempering and infinite swapping algorithms

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Parallel tempering, or replica exchange, is a method for simulating complex systems and one of the workhorses for computational tasks in the physical sciences. The idea is to run parallel simulations at different “temperatures” - either corresponding to a physical temperature or a parameter in the system that plays a similar role - and at a given swap rate exchange configurations between the parallel simulations. From the perspective of large deviations it is optimal to send the swap rate to infinity and it is possible to construct a corresponding simulation scheme, known as infinite swapping. In this talk we will describe how empirical measure large deviations can be used for a detailed study of the infinite swapping limit. From the large deviations rate function and associated stochastic control problems we gain a better understanding of certain properties of the corresponding simulation algorithms. Time permitting we will also discuss some potential applications of the infinite swapping methodology in situations where rare-event sampling is an issue.

This is based on joint work with J. D. Doll and P. Dupuis, and on-going work with H. Hult and C. Ringqvist.

References


Generalized space time fractional equation and the related stochastic processes

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We consider space-time fractional equations of the form

$$\sum_{j=1}^{m} \lambda_j \frac{\partial^{\nu_j}}{\partial t^{\nu_j}} w(x, t) = -c^2 (-\Delta)^{\beta} w(x, t) \quad x \in \mathbb{R}^n$$  \hspace{1cm} (2.5)

where \(\lambda_j > 0, c > 0, 0 < \nu_j < 1, (-\Delta)^{\beta}, 0 < \beta \leq 1\) is the fractional Laplacian and \(\frac{\partial^{\nu_j}}{\partial t^{\nu_j}}\) are time-fractional derivatives in the sense of Caputo.

We show that the solution to the above equation coincides with the law of a time-changed isotropic stable process with characteristic function

$$\mathbb{E} e^{i \xi \cdot S_{t}^{\nu_1, \ldots, \nu_m}(t)} = e^{-t ||\xi||^{2\beta}}$$

where the time is represented

$$L^{\nu_1, \ldots, \nu_m}(t) = \inf \left\{ s : \sum_{j=1}^{m} \lambda_j \frac{1}{\nu_j} H^{\nu_j} (s) \geq t \right\}$$  \hspace{1cm} (2.6)

with \(H^{\nu_j}\) independent stable subordinators of order \(\nu_j\). Equation (2.5) includes the space-time fractional telegraph equation as a special case which for \(n = 1\) reads

$$\frac{\partial^{2\nu}}{\partial t^{2\nu}} u + 2 \frac{\partial^\nu}{\partial t^\nu} u = c^2 \frac{\partial^{2\beta}}{\partial |x|^{2\beta}} u$$  \hspace{1cm} (2.7)

where \(\frac{\partial^{2\beta}}{\partial |x|^{2\beta}}\) is the Riesz fractional operator. In this case one can write down the Fourier transform of the fundamental solution as

$$U(\xi, t) = \frac{1}{2} \left[ \left( 1 + \frac{\lambda}{\sqrt{\lambda^2 - c^2 |\xi|^2}} \right) E_{\nu, 1}(\eta_1 t^\nu) \right.$$ \hspace{1cm} (2.8)

$$+ \left( 1 - \frac{\lambda}{\sqrt{\lambda^2 - c^2 |\xi|^2}} \right) E_{\nu, 1}(\eta_2 t^\nu) \right]

where \(E_{\nu, 1}\) is the Mittag-Leffler function, \(\eta_1 = -\lambda + \sqrt{\lambda^2 - c^2 |\xi|^2}\) and \(\eta_2 = -\lambda - \sqrt{\lambda^2 - c^2 |\xi|^2}\).

For \(\nu = \frac{1}{2}, \beta = 2\), the function \(\eta_1, \eta_2\) can be regarded as the distribution of

$$T(|B(t)|) \overset{d}{=} B \left( c^2 L^{\frac{\nu}{2}}(t) \right)$$  \hspace{1cm} (2.9)

where on one side the telegraph process \(T\) is composed with a reflecting Brownian motion and on the other side we have a time-changed Brownian motion with

$$L^{\frac{\nu}{2}}(t) = \inf \left\{ s : s + (2\lambda)^2 H^{\nu}(s) \geq t \right\}

Further special cases are also illustrated.

References

Diffusion associated with the zeros of the planner Gaussian analytic function.

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It is known that the point process given by the zeros of the planner Gaussian analytic function is translation invariant, and has a rigidity stricter than the Ginibre point process. Indeed, if we fix the outside configuration of the ball, then the mean position of the particles inside is deterministic as well as the number of the particles inside. As a result, the distribution of particles inside does not have any local density with respect to the full Lebesgue measure. In this talk we construct unlabeled diffusion reversible to this point process. We use the Dirichlet form technology for the proof. The lack of the local density requires a new technique of the proof.

Discrete approximations of determinantal measures and tail triviality

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A determinantal point process $\mu$ on $S$ is a probability measure on the configuration space $S = \text{Conf}(S)$ of which correlation functions are given by the determinant of a kernel $K(x, y)$ on $S$.

We say $\mu$ is tail trivial if $\mu(A) \in \{0, 1\}$ for all $A \in \mathcal{T}(S)$, where $\mathcal{T}(S)$ is the tail $\sigma$-field of $S$. If the space is discrete, tail triviality has been proved by Shirai-Takahashi [4] for Spec($K$) $\subset (0, 1)$, and Russell Lyons [1] for Spec($K$) $\subset [0, 1]$. We prove tail triviality of $\mu$ when $S$ is continuum [2].

Let $S$ be a locally compact, complete, and separable metric space equipped with Radon measure $m$. In this talk we assume that $(K, m)$ satisfies following conditions (A.1): (1) $K$ is Hermitian symmetric, (2) of locally trace class, and (3) Spec($K$) $\subset [0, 1]$, where $Kf(x) = \int_S K(x, y)f(y)m(dy)$. It is known that if $(K, m)$ satisfies (A.1), then there uniquely exists the determinantal point process $\mu$ which correlation functions are given by determinants of $K$ [3, 5]. We call this $\mu$ as the $(K, m)$-DPP on $S$.

Let $\{\Delta(l); l \in \mathbb{N}\}$ be a sequence of partitions such that each component of $\Delta(l)$ is subdevided in two as $l$ grows.

Theorem. If $\{\Delta(l); l \in \mathbb{N}\}$ satisfies (A.2), then $\mu$ is tail trivial.

(A.2) : $\sigma[\bigcup_{l \in \mathbb{N}} \Delta(l)] = \mathcal{B}(S)$, where $\mathcal{B}(S)$ is the Borel $\sigma$-field of $S$.

In this proof, we construct a discretization of $\mu$ which is tail trivial and inherit tail triviality to $\mu$ by martingale convergence theorem. Firstly we take a sub $\sigma$-field:

$$\mathcal{G}_i = \sigma\{s \in S; s(A) = n\}; A \in \Delta(l), n \in \mathbb{N}.$$

If the regular conditional probability $\mu(\cdot | \mathcal{G}_i)$ is tail trivial, then we deduce that $\mu$ is tail trivial from the martingale convergence theorem. Instead of $\mu(\cdot | \mathcal{G}_i)$, we will show tail triviality of $\mu_i = \mu|_{\mathcal{G}_i}$: restriction of $\mu$ on $\mathcal{G}_i$. Secondly, to prove tail triviality of $\mu_i$, we introduce the discretization as a kind of Fourier transform of $\mu$. We construct an orthonormal basis $\mathcal{F}(l) = \{f_i; i \in \mathbb{N}(l)\}$ in $L(S, m)$, here $\mathbb{N}(l)$ is the index set. $\mathcal{F}(l)$ gives a Fourier transform of $K$, so Fourier coefficients $\{K_{ij}(i, j); i, j \in \mathbb{N}(l)\}$ can be regarded as a kernel on $\mathbb{N}(l)$. From the result due to [3, 5], there exists the $(K_l, \lambda)$-DPP : $\nu_l$, where $\lambda$ is the counting measure on $\mathbb{N}(l)$. $\nu_l$ is a determinantal point process on $\mathbb{N}(l)$ and is tail trivial because $\mathbb{N}(l)$ is a discrete set. Moreover, following Parseval’s like equation holds.

Lemma. Let $f_{i}^{\mu}$ and $f_{i}^{\nu}$ be correlation functions of $\mu$ and $\nu$ in each other.

For $\Lambda = (A_1, \ldots, A_n)$, we set $\mathbb{I}_i(\Lambda) = \mathbb{I}_i(A_1) \times \cdots \times \mathbb{I}_i(A_n)$ where $\mathbb{I}_i(A) = \{i \in \mathbb{N}(l); \text{supp} f_i \subset A\}$. 

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If $A_m \in \Delta(l)$ for $m = 1, \ldots, n$, then we have
\[
\int \rho_m^n(x_1, \ldots, x_n)m^n(dx) = \sum_{i \in \ell(y)} \rho_n^i(i_1, \ldots, i_n).
\]

Finally this implies $\mu_l = \nu_l \circ \Pi^{-1}$, here $\Pi$ is a projection. Hence $\mu_l$ inherits tail trivial of $\nu_l$ and $\mu$ also does.

References


Modularity in several random graph models

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One important property of many complex networks is their community structure, that is, the organization of vertices in clusters, with many edges joining vertices of the same cluster and comparatively few edges joining vertices of different clusters. In social networks communities may represent groups by interest, in citation networks they correspond to related papers, in the Web communities are formed by pages on related topics, etc.

Modularity is at the same time a global criterion to define communities, a quality function of community detection algorithms, and a way to measure the presence of community structure in a network. Modularity was first introduced by Newman and Girvan in [6]. Since then, many popular and applied algorithms used to find clusters in large data-sets are based on finding partitions with high modularity.

The main idea behind modularity is to compare the actual density of edges inside communities with the density one would expect to have if the vertices of the graph were attached at random, regardless of community structure. Formally, for a given partition $A = \{A_1, \ldots, A_k\}$ of the vertex set $V(G)$, let
\[
q_A = \sum_{A \in \mathcal{A}} \left( \frac{e(A)}{|E(G)|} - \frac{(\sum_{v \in A} \deg(v))^2}{4|E(G)|^2} \right),
\]
where $e(A) = |\{uv \in E(G) : u, v \in A\}|$ is the number of edges in the graph induced by the set $A$. The first term, $\sum_{A \in \mathcal{A}} \frac{e(A)}{|E(G)|}$, is called the edge contribution, whereas the second one, $\sum_{A \in \mathcal{A}} \frac{(\sum_{v \in A} \deg(v))^2}{4|E(G)|^2}$, is called the degree tax. It is easy to see that $q_A$ is always smaller than one. Also, if $A = \{V(G)\}$, then $q_A = 0$. The modularity $q^*(G)$ is
\[
q^*(G) = \max_A q_A(G).
\]
If $q^*(G)$ approaches 1 (which is the maximum), we observe a strong community structure; conversely, if $q^*(G)$ is close to zero, we are given a graph with no community structure.
Unfortunately, modularity is not a well studied parameter for the existing random graph models, at least from a rigorous, theoretical point of view. In this work, we investigate modularity in random \(d\)-regular graphs, the preferential attachment model, and the spatial preferential attachment model. The detailed results can be found in [7].

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**Optimal survival strategy for branching brownian motion in a Poissonian trap field**

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We study a branching Brownian motion (BBM) \(Z\) evolving in \(\mathbb{R}^d\), where a uniform field of Poissonian traps is present. Each trap is a ball with constant radius. Using classical results on the convergence of the speed of branching Brownian motion, we find upper bounds on the population size of \(Z\) given that it avoids the trap field up to time \(t\). The result is stated so that it gives an ‘optimal survival strategy’ for \(Z\). On the way to its proof, we establish a result of independent interest, which gives an upper bound on the survival probability of a BBM in a large class of random trap fields. As corollaries of our primary result concerning the population size, we prove several other optimal survival strategies concerning the range of \(Z\), and the size and location of trap-free regions in \(\mathbb{R}^d\).

**References**


The geometry of functionally generated portfolios

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Functionally generated portfolios were introduced by Robert Fernholz to outperform a capitalization-weighted index (such as S&P500) in the long term. These are generalizations of popular statistical arbitrage portfolios such as pair trading. Natural questions about such portfolios such as uniqueness and optimal frequency of rebalancing can be answered by investigating the underlying convex geometry and a related optimal transport problem. We will also discuss how concentration of measure in high dimension can significantly boost the performance of such portfolios.

Continuous state branching processes in a Lévy random environment.

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In this talk we will discuss the construction of a continuous state branching process with immigration in a Lévy random environment. In the case of stable CB-process, we are going to describe the asymptotic behaviour of the extinction and explosion probabilities. This will allow us to analyse the process conditioned to extinction and the process conditioned to never been absorb.

Acknowledgement: Sandra Palau acknowledge support from CONACyT-MEXICO Grant 351643 and form the Royal Society under a Newton International Fellowship NF160591. J.C. Pardo acknowledge support from the Royal Society

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Fluctuations of Omega-killed spectrally negative Lévy processes

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In this talk we present the solutions of so-called exit problems for a (reflected) spectrally negative one-dimensional Lévy process exponentially killed with killing intensity depending on the present state of the process. We will also analyze respective resolvents. All identities are given in terms of new generalizations of scale functions. Particular cases concern $\omega(x) = q$ when we derive classical exit problems and $\omega(x) = q1_{[a,b]}(x)$ producing Laplace transforms of occupation times of intervals until first passage times. We will show how derived results can be applied to find bankruptcy probability in so-called Omega model, where bankruptcy occurs at rate $\omega(x)$ when the surplus Lévy process process is at level $x < 0$. Finally, we demonstrate how to get some exit identities for a spectrally positive self-similar Markov processes. The main idea of all proofs relies on classical fluctuation identities for Lévy process, the Markov property and some basic properties of a Poisson process. The talk is based on [1].

References


TASEP in continuous inhomogeneous space

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We discuss a version of TASEP in continuous time and space, in which the space can be equipped with arbitrary inhomogeneity. This system is integrable owing to a new connection with Schur measures. This leads to limit shape and KPZ-type fluctuation results. We also discuss other variants of this system, including a discrete version (which can be mapped to a directed percolation model) and a $q$-deformation.

Homogenization of random parabolic operators. Diffusion approximation

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We consider a Cauchy parabolic problem

$$\frac{\partial}{\partial t}u^\varepsilon = \text{div} \left[ a\left( \frac{x}{\varepsilon}, \frac{t}{\varepsilon^2} \right) \nabla u^\varepsilon \right], \quad (x, t) \in \mathbb{R}^d \times (0,T),$$

$$u^\varepsilon(x,0) = \varphi(x)$$
with periodic in spatial variables and random stationary ergodic in time coefficients $a^{ij}(z,s)$, and study the limit behaviour of its solution, as positive parameter $\varepsilon$ tends to zero. We assume that the matrix $a(z,s)$ satisfies the uniform ellipticity conditions, and that $\varphi$ is a Schwartz class function.

It is known (see[1]) that for any $\alpha > 0$ the solution $u^\varepsilon$ converges to the solution of the homogenized Cauchy problem

$$\frac{\partial}{\partial t} u^0 = \text{div} \left[ a^{\text{eff}} \nabla u^0 \right], \quad (x,t) \in \mathbb{R}^d \times (0,T),$$

$$u^0(x,0) = \varphi(x)$$

with constant deterministic coefficients. Our goal is to describe the asymptotic behaviour of the difference $u^\varepsilon - u^0$.

We suppose that $\alpha \leq 2$ and consider separately the cases $\alpha = 2$ and $\alpha < 2$.

Denote by $\rho(\cdot)$ the maximum correlation coefficient of the matrix $a$. In the case $\alpha = 2$ we have

**Theorem.** (see [2]) Assume that $\int_0^\infty \rho(s)ds < +\infty$. Then the normalized difference $\varepsilon^{-1}(u^\varepsilon - u^0)$ converges in law, as $\varepsilon \to 0$, in $L^2(\mathbb{R}^d \times (0,T))$ to a solution of the following SPDE

$$dt u^0 = \text{div} \left[ a^{\text{eff}} \nabla u^0 + \Xi \frac{\partial^3}{\partial x^3} u^0 \right] dt + \Lambda \frac{\partial^2}{\partial x^2} u^0 dW_t,$$

$$U^0(x,0) = 0,$$

(2.11)

where $\Xi = \{\Xi^{ijk}\}$ and $\Lambda = \{\Lambda^{ijkl}\}$ are constant tensors and $W$ is a standard $d^2$-dimensional Wiener process.

If $\alpha < 2$ then we have

**Theorem.** Assume that $\int_0^\infty \rho(s)ds < +\infty$. Then there exist smooth functions $u^1(x,t), \ldots, u^{J_0}(x,t)$ with $J_0$ being equal to the integer part of $\frac{1}{2-\alpha}$, such that

$$U^\varepsilon = \varepsilon^{-\alpha/2} \left( u^\varepsilon - u^0 - \sum_{j=1}^{J_0} \varepsilon^{(2-\alpha)} u^j \right)$$

converges in law, as $\varepsilon \to 0$, in $L^2(\mathbb{R}^d \times (0,T))$ to a solution of the SPDE that has the same form as (2.11).

**References**


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A probabilistic approximation of the Cauchy problem solution for an evolution equation with the differential operator of the order greater than 2

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It is well known that a solution of the Cauchy problem for the heat equation

$$\frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial x^2}, \quad u(0,x) = \varphi(x)$$

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can be represented as an integral with respect to the Wiener process distribution density $\mu(t, y)$

$$u(t, x) = \mathbb{E}\varphi(x - w(t)) = \int_R \varphi(x - y)\mu(t, y)dy$$  \hspace{1cm} (2.12)

where $w(t)$ is a standard Wiener process.

If we consider the Cauchy problem for an evolution equation with a differential operator of the order $m > 2$

$$\frac{\partial u}{\partial t} = \frac{c_m}{m!} \frac{\partial^m u}{\partial x^m}, \quad u(0, x) = \varphi(x)$$  \hspace{1cm} (2.13)

where

$$c_m = \begin{cases} \pm 1, & m = 2k + 1, \\ (-1)^{k+1}, & m = 2k, \end{cases}$$

then since a fundamental solution of the equation (2) is not a density of a probabilistic measure, one can not hope to derive a formula similar to (1).

Nevertheless we have constructed a probabilistic approximation of a solution of the Cauchy problem (2) in the Sobolev space $W^{l+m+1}_2(R)$, $l > 0$ based on generalized function theory methods and point process theory.

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References


In russian.

Mean-field behaviour of pathogen systems subject to balancing selection and reinfection

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Motivated from DNA data of the herpesvirus CMV we model the evolution of a two-type CMV population by a Markov jump process $X^{N,M}$ representing the type frequencies in $M$ hosts, each infected with a viral population of size $N$.

Evolution is driven by three factors: a) viral reproduction, b) host replacement and c) reinfection. Within hosts, viruses reproduce subject to balancing selection, as we assume that viruses profit from infecting a host with more than one type. Whenever a host dies, it is replaced by a new, so far uninfected host, which instantly suffers a primary infection by a randomly chosen infected host. At primary infection the host is infected with a single type chosen randomly according to the type frequencies in the infecting host. This leads to a jump to type frequency 0 or 1 in the primary infected host. At reinfection a single virion in the reinfecting host is replaced by a randomly chosen virion transmitted from the infecting host. Only some of these reinfections are “successful”, in the sense that reinfection takes the type frequency of a beforehand single-type infected host to the (quasi-)equilibrium.

Now assume that viral reproduction is much faster than host replacement, and successful reinfection and host replacement act on the same time scale. Then in the limit of a large viral population ($N \to \infty$) and large host population ($M \to \infty$) we show by using near-critical branching processes that $X^{N,M}$ converges under moderate selection to a mean-field limit, in which in every individual host type frequencies take only values in 0,1 and the equilibrium frequency of the balancing selection.

This is work in progress.
Asymptotic exponentiality of the first exit time of the shiryaev–roberts diffusion with constant positive drift

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We consider the first exit time of a Shiryaev–Roberts diffusion with constant positive drift from the interval $[0, A]$ where $A > 0$. We show that the moment generating function (Laplace transform) of a suitably standardized version of the first exit time converges to that of the unit-mean exponential distribution as $A \to +\infty$. The proof is explicit in that the moment generating function of the first exit time is first expressed analytically and in a closed form, and then the desired limit as $A \to +\infty$ is evaluated directly. The result is of importance in the area of quickest change-point detection, and its discrete-time counterpart has been previously established—although in a different manner—by Pollak & Tartakovsky (2009).

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References


An invariance principle for branching diffusions in bounded domains

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We study branching diffusions in a bounded domain $D$ of $\mathbb{R}^d$ in which particles are killed upon hitting the boundary $\partial D$. It is known that any such process undergoes a phase transition when the branching rate $\beta$ exceeds a critical value: a multiple of the first eigenvalue of the generator of the diffusion. We investigate the system at criticality, and prove an asymptotic for the probability of survival up to large times. We show further that the genealogical tree associated with such a critical process converges to Aldous’ Continuum Random Tree under appropriate rescaling. The result holds under only a mild assumption on the domain, and is valid for all branching mechanisms with finite variance, and a general class of diffusions.

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Gittins Type Index for Randomly Evolving Graphs

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We deal with randomly evolving directed tree. Initially a finite set of independent trials (edges of the tree) is available. If a Decision Maker (DM) choose to test a specific trial she receives a reward, and with some probability, the process of testing is terminated or the tested trial becomes unavailable but some random finite set (possibly empty) of new independent trials is added to the set of initial trials, and so on. On each step she can either stop the process of testing or continue. Her goal is to select an order to test trials and an exit (stopping) time to maximize the expected total reward. We prove that an index can be assigned to each possible trial and optimal strategy uses on each step the trial with maximal index between available ones. We present a recursive procedure with a transparent interpretation to calculate the index. We discuss the connection between introduced index and Gittins index.

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On Covering Monotonic Paths with Simple Random Walks

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In this paper we study the probability that a \(d\) dimensional simple random walk (or the first \(L\) steps of it) covers each point in a nearest neighbor path connecting 0 and the boundary of an \(L_1\) ball. We show that among all such paths, the one that maximizes the covering probability is the monotonic increasing one that stays within distance 1 from the diagonal. As a result, we can obtain an exponential upper bound on the decaying rate of covering probability of any such path when \(d \geq 4\).

Integro-local limit theorems for multidimensional compound renewal processes

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Let \((\tau, \zeta), (\tau_1, \zeta_1), (\tau_2, \zeta_2), \cdots\) be a sequence of i.i.d. random vectors in \(\mathbb{R} \times \mathbb{R}^d\), \(\tau > 0\),

\[ T_0 := 0, \quad T_n := \sum_{j=1}^{n} \tau_j, \quad Z_0 := 0, \quad Z_n := \sum_{j=1}^{n} \zeta_j; \]

\[ \eta(t) := \max\{k \geq 0 : T_k < t\}, \quad \nu(t) := \min\{k \geq 0 : T_k \geq t\}. \]

The compound renewal processes \(Z(t), Y(t)\) for the sequence \((\tau_j, \zeta_j), j \geq 1\), are defined as

\[ Z(t) := Z_{\eta(t)}, \quad Y(t) := Z_{\nu(t)}, \quad t \geq 0. \]
Let the Cramér moment condition for \((\tau, \zeta)\) hold. For a vector \(x = (x_1, \cdots, x_d) \in \mathbb{R}^d\) put
\[
\Delta |x| := [x_1, x_1 + \Delta] \times [x_2, x_2 + \Delta] \times \cdots \times [x_d, x_d + \Delta], \quad \Delta > 0.
\]
We establish the exact asymptotics for the probabilities
\[
P(Z(t) \in \Delta|x|), \quad P(Y(t) \in \Delta|x|), \text{ as } t \to \infty
\]
in the range of normal and large deviations.

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### Critical controlled branching processes

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In this talk we deal with the class of controlled branching processes (CBPs). Adopting the terminology of population dynamics, these are discrete-time stochastic processes that are used to describe the evolution of populations where each individual lives for a unit of time and is replaced by a random number of offspring, and where a control on the population size in each generation is needed. This control is made by determining the number of individuals with reproductive capacity at each generation through a random mechanism. The novelty of adding a random control mechanism to the branching notion allows to model a great variety of random migratory movements (immigration, emigration, or even both depending on the generation sizes). Emulating the Bienaymé–Galton–Watson process classification, there exists a threshold parameter, called the asymptotic mean growth rate, which determines the behaviour of a CBP in relation to its extinction. Thus, we shall term a CBP as subcritical, critical, or supercritical depending on whether such a parameter is less than, equal to, or greater than one. We present a survey of the main results obtained until now about the extinction problem, the limit behaviour and the statistical inference arising from this model, focusing our attention in the critical case.
On long term investment optimality

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We study the problem of optimal long term investment with a view to beat a benchmark for a diffusion model of asset prices. Two kinds of objectives are considered. One criterion concerns the probability of outperforming the benchmark and seeks either to minimise the decay rate of the probability that a portfolio exceeds the benchmark or to maximise the decay rate that the portfolio falls short. The other criterion concerns the growth rate of the risk-sensitive utility of wealth which has to be either minimised, for a risk-averse investor, or maximised, for a risk-seeking investor. It is assumed that the mean returns and volatilities of the securities are affected by an economic factor, possibly, in a nonlinear fashion. The economic factor and the benchmark are modelled with general Itô differential equations. The results identify optimal portfolios and produce the decay, or growth, rates. The portfolios have the form of time-homogeneous functions of the economic factor. Furthermore, a uniform treatment is given to the out- and under-performance probability optimisation as well as to the risk-averse and risk-seeking portfolio optimisation. It is shown that there exists a portfolio that optimises the decay rates of both the outperformance probability and the underperformance probability. While earlier research on the subject has relied on the techniques of stochastic optimal control and dynamic programming, in this contribution the quantities of interest are studied directly by employing the methods of the large deviation theory. The key to the analysis is to recognise the setup in question as a case of coupled diffusions with time scale separation, with the economic factor representing "the fast motion".

Uniqueness for Measure-Valued Equations of Nonlinear Filtering for Stochastic Dynamical Systems with Lévy Noises

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In the article, Zakai and Kushner-Stratonovich equations of the nonlinear filtering problem for a non-Gaussian signal-observation system are considered. Moreover, we prove that under some general assumption, the Zakai equation has pathwise uniqueness and uniqueness in joint law, and the Kushner-Stratonovich equation is unique in joint law.

References

Pathwise Differentiability of Obliquely Reflected Diffusions

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Differentiability of flows and sensitivity analysis are classical topics in dynamical systems. However, the analysis of these properties is challenging for constrained stochastic processes such as obliquely reflected diffusions, which arise in a variety of applications ranging from the study of interacting Brownian particle systems and Atlas models to biochemical reaction networks and queueing networks. The challenge arises due to the discontinuous dynamics at the boundary of the domain, and is further complicated when the boundary is non-smooth. We show that the study of both differentiability of flows and sensitivities of constrained processes in convex polyhedral domains are reduced to the study of directional derivatives of an associated map, called the Skorokhod map, and we introduce an axiomatic framework to characterize these directional derivatives. In addition, we establish pathwise differentiability of a large class of reflected Brownian motions in convex polyhedral domains and show that they can be described in terms of certain constrained stochastic differential equations with jumps. We also discuss applications to sensitivity analysis.

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References


Large Deviations for Mean-Field Games

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Consider a dynamic symmetric game with $n$ players in which both the cost structure and strategy of each player depends on its own state and on the state of the other players only through the empirical distribution of their states. The Nash equilibria of such symmetric $n$-player games are hard to analyze or even compute, but their limit, as the number of players goes to infinity, can be characterized in terms of a certain stochastic differential game with infinitely many players, referred to as a mean-field game. For a certain class of games, for which the mean-field game has a unique equilibrium, we show how properties of solutions to an infinite-dimensional partial differential equation associated with the mean-field game can be used to establish central limit theorems and large deviation principles for the sequence of empirical measures of Nash equilibria in $n$-player games.

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Some stochastic models for seasonal rainfall at fine time-scales

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Stochastic point process models have been widely used to model rainfall time series. Doubly stochastic Poisson processes provide a rich class of models for analysing fine time-scale rainfall data. Models of this type have been used by several authors to describe fine-scale rainfall characteristics at a single site as well as at multiple sites. Ramesh et al. (2013) developed a class of multisite models for analysing tipping-bucket rainfall data recorded over a number of stations in a catchment area.

In this paper, we extend the univariate class of models for fine time-scale rainfall to accommodate seasonality and study a number of seasonal doubly stochastic Poisson process models. This includes models incorporating atmospheric covariates in the analysis. The application of these models is illustrated in the modelling of sub-hourly rain gauge data from England. One of the advantages of this class of models, when compared with similar models, is that their likelihood function can be calculated in a tractable form suitable for numerical optimisation. This allows us to use the maximum likelihood approach to estimate the parameters of the proposed stochastic models. We use some of the second-order properties of the fine-scale rainfall aggregations in discrete time intervals for model assessment.

References

Markov Switching asymmetric GARCH Model: Stability and Forecasting

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A new dynamic Markov switching asymmetric GARCH model is proposed that the volatility in states are coupled with some logistic smooth transition based on the size and sign of previous observation. This model is called Markov switching smooth transition GARCH model, in summary MS-STGARCH, and this model is defined as

$$y_t = \varepsilon_t \sqrt{H_{Z_t,t}}$$

where $$\{\varepsilon_t\}$$ are iid standard normal variables, $$\{Z_t\}$$ is an irreducible and aperiodic Markov chain on finite state space $$E = \{1, 2, \cdots, K\}$$ with transition probability matrix $$P = [p_{ij}]_{K \times K}$$, where the probability transition $$p_{ij} = p(Z_t = j\mid Z_{t-1} = i)$$, $$i, j \in \{1, \cdots, K\}$$, and stationary probability measure $$\pi = (\pi_1, \cdots, \pi_K)'$$.

Also given that $$Z_t = j$$, $$H_{j,t}$$ (the conditional variance of regime $$j$$) is driven by

$$y_{t+1} = a_0 + y_{t-1}^2 (d_{j,t} + b_j H_{j,t-1}) + w_{j,t}$$

where

$$d_{j,t} = a_{1j}(1 - w_{j,t-1}) + a_{2j} w_{j,t-1}$$

and each of the weights ($$w_{j,t}$$) is a logistic function of the past observation as

$$w_{j,t} = \frac{1}{1+\exp(-\gamma_j y_{t-1})}$$

This consideration cause to have better forecasts in many financial time series where different level of volatilities are presented and there is asymmetric effects of negative and positive shocks. A Bayesian strategy through Gibbs and griddy Gibbs sampling is used to estimate the parameters. For some part of the S&P500 indices the out-of-sample forecasting performance of one day ahead volatility and value at risk of the proposed model are evaluated where our model outperforms the competing models for in-sample fit and out-of-sample forecasts. Also the asymptotic behavior of the second moment is investigated and an upper bound is evaluated.

**Theorem.** The conditional variance of MS-STGARCH model is given by

$$Var(Y_t|Z_{t-1}) = \sum_{i=1}^{K} \alpha_i^{(t)} H_{i,t} = \sum_{i=1}^{K} \alpha_i^{(t)} (a_0 + a_{1i} y_{t-1}^2 (1 - w_{i,t-1})) + a_{2i} y_{t-1}^2 w_{i,t-1} + b_i H_{i,t-1}$$

where $$\alpha_i^{(t)}$$ is obtained recursively by

$$\alpha_i^{(t)} = \sum_{m=1}^{K} \frac{f(y_{t-1}|Z_{t-1} = m, \mathcal{I}_{t-2}) p(Z_{t-1} = m|\mathcal{I}_{t-2}) p_{m,i}}{\sum_{m=1}^{K} f(y_{t-1}|Z_{t-1} = m, \mathcal{I}_{t-2}) p(Z_{t-1} = m|\mathcal{I}_{t-2})}$$

**References**


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Rough differential equations with unbounded drift and random dynamical systems

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We consider rough differential equations of the form

\[ dy = b(y) \, dt + \sigma(y) \, dx, \]  

(2.16)

\( x \) being a rough path in the sense of T. Lyons, with a possibly unbounded drift term \( b \). It turns out that the usual one-sided growth conditions are not sufficient to guarantee non-explosion of the solution in finite time. We provide a further condition under which non-explosion can be assured. Allowing for an unbounded drift becomes important when studying the long time behaviour of a solution to (2.16). If the rough path is random, a powerful tool for this study is the theory of random dynamical systems. We explain the connection and discuss possible applications.

This is joint work with I. Bailleul (Rennes) and M. Scheutzow (Berlin).

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References


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The genealogical structure of near-critical branching processes

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Take a continuous-time Galton-Watson tree. If the system survives until a large time \( T \), then choose \( k \) particles uniformly from those alive. What does the ancestral tree drawn out by these \( k \) particles look like? Some special cases are known but we give a more complete answer, including an interesting and explicit universal scaling limit in near-critical cases. Some extensions to branching Brownian motion (work in progress) will be briefly mentioned in order to meet the topic of the session.

References

Stochastic flocking or how much coherent is the (random) flight of wild birds

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In this talk we first introduce the notion of flocking (self-organisation) for dynamical systems like the Cucker-Smale model. One can observe this type of global coherence phenomenon of the motion for example by the flight of wild birds. Then we discuss several stochastic versions of such dynamical systems, as well as their asymptotics for large size or in time.

Exponential convergence rates for stochastically ordered Markov processes with random initial conditions

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In this paper we extend the results of Lund, Meyn, and Tweedie (1996) on exponential convergence rates for stochastically ordered Markov processes to allow for a random initial condition that is also stochastically ordered. We find an explicit exponential convergence rate for an M/M/1 queue beginning in equilibrium and then experiencing a perturbation of its arrival or departure rates. In order to show the convergence bound we also prove a result on hitting times of differences of independent Poisson processes, known as Skellam processes. Finally, we comment on the applicability of our result to other stochastically increasing processes, such as Jackson networks experiencing a change in external arrival rates.

References


Nodal lengths in Wiener chaos

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In this talk we are interested in the local behavior of Gaussian Laplacian eigenfunctions on two-dimensional Riemannian manifolds. In particular, we investigate the asymptotic behavior of their nodal length in the case of the standard torus, the unit sphere and the Euclidean plane.

The main tool is the chaotic expansion for the nodal length that allows to compute its mean, asymptotic variance and limiting distribution (obtaining also rates of convergence in some probability metric). Indeed, the “single-chaotic-component dominating” phenomenon appears in all the above mentioned models, making easier the investigation of the asymptotic nodal properties.

While for the asymptotic variance (roughly) the same kind of results is obtained, the limiting distribution instead is deeply affected by the geometry of the underlying manifold. To be clearer, a non-Central Limit Theorem is proved for toral nodal lengths, while Gaussian fluctuations are expected for the other two models.

This talk is based on joint works with D. Marinucci, I. Nourdin, G. Peccati and I. Wigman.

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Towards the Einstein relation for the Mott variable-range hopping and other random walks

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The Mott Variable-Range Hopping model is considered in Physics as an accurate representation of electrical conduction in semiconductors. From the mathematical point of view, it represents a prominent example of reversible long-range random walks on random point processes, which generalize in several ways the classical random conductance model on the lattice. We ask ourselves how an external field influences the limiting velocity of the walk: So far, only very few models of biased random walks with trapping mechanisms have been rigorously studied. We will also give a precise control of the invariant measure for the process from the point of view of the particle and explain how this control should lead to the proof of the Einstein Relation for this and related models.

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References

Local single ring theorem

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In the first part of this talk, I will discuss some recent results on local laws and rigidity of eigenvalues for additive random matrix models. In the second part, I will explain how these results can be used to derive the optimal convergence rate of the empirical eigenvalue distribution in the Single Ring Theorem.

References


Entropy estimation for stationary $m$-dependent sequences

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We study estimation of certain integral functionals for the marginal densities of stationary $m$-dependent sequences (e.g., Rényi entropy for the marginal distribution). The Rényi entropy is a generalization of the Shannon entropy and is widely used in mathematical statistics and applied sciences for quantifying the uncertainty in a probability distribution. Similar problems for independent samples are considered in our earlier papers, e.g., [1],[2]. The $U$-statistic estimators under study are based on the number of $\varepsilon$-close vector observations in the corresponding sample. A variety of asymptotic properties for these estimators are obtained (e.g., consistency, asymptotic normality, Poisson convergence). The results can be used in diverse statistical and computer science problems whenever the conventional independence assumption is too strong. One of the obtained applications is a certain consistency property for sample entropy, widely used as a common index to quantify the complexity of time series in various fields (e.g., medicine, financial mathematics, [3]).

References

Pesin’s formula for isotropic Brownian flows

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Pesin’s formula is a relation between the entropy of a dynamical system and its positive Lyapunov exponents. This formula was first established by Pesin in the late 1970s for some deterministic dynamical systems acting on a compact Riemannian manifold. Later were obtained plenty of generalizations of it. For example, different authors have proved the formula for some random dynamical systems, or have relaxed the condition of state space compactness. Nevertheless, it has never been obtained for dynamical systems with invariant measure, which is infinite. The problem is that if invariant measure is infinite, then the notion of entropy becomes senseless. Invariant measure of isotropic Brownian flows is the Lebesgue measure on $\mathbb{R}^d$, which is, clearly, infinite. Nevertheless, we define entropy for such a kind of flows using their translation invariance. Then we study the analogue of Pesin’s formula for these flows using the defined entropy.

On the $r$-colorability threshold in a random hypergraph

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The talk deals with estimating the probability threshold for $r$-colorability property in a random hypergraph. Let $H(n, k, p)$ denote the classical binomial model of a random $k$-uniform hypergraph: every edge of a complete $k$-uniform hypergraph on $n$ vertices is included into $H(n, k, p)$ as an edge independently with probability $p \in (0, 1)$.

We study the question of estimating the probability threshold for the $r$-colorability property of $H(n, k, p)$. Recall that a hypergraph is $r$-colorable if there exists a vertex coloring with $r$ colors without monochromatic edges. It is well known that for fixed $r \geq 2$ and $k \geq 2$, this threshold appears in a sparse case when the expected number of edges is a linear function of $n$: $p \binom{n}{k} = cn$ for some fixed $c > 0$.

The following result gives a new lower bound for the $r$-colorability threshold.

**Theorem.** Let $k \geq 4$, $r \geq 2$ be integers and $c > 0$. Then there exist absolute constants $C > 0$ and $d_0 > 0$ such that if $\max(r, k) > d_0$ and

$$c < r^{k-1} \frac{\ln r}{2} - \frac{r - 1}{r} - C \cdot \frac{k^2 \ln r}{r^{k/3 - 1}},$$

(2.17)

then

$$\Pr\left( H(n, k, cn/k) \text{ is } r\text{-colorable} \right) \to 1 \text{ as } n \to +\infty.$$  

Theorem improves the previous result from [1] and provides a bounded gap with the known upper bound. The estimate (2.17) is only $\frac{r-1}{r} + O\left(\frac{k^2 \ln r}{r^{k/3-1}}\right)$ less than the upper bound from [1]. If the value of the parameter $r$ is much greater than $k$, then a slightly better result is known [2]. The proof is based on a new approach to the second moment method.

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A series of well-known problems of theoretical computer science and combinatorics (e.g. kSAT problem, problem concerning estimating the Ramsey numbers etc.) can be formulated in terms of abstract hypergraph coloring problems. A hypergraph is a pair $H = (V, E)$ where $V$ is a finite set, called the vertex of the hypergraph, and $E$ is a collection of subsets of $V$ which are called the edges of the hypergraph. A hypergraph is said to be $k$-uniform if every of its edges consists of exactly $k$ vertices. A degree of an edge is the number of other edges that intersect this edge. The maximum edge degree is denoted by $\Delta(H)$. A vertex coloring is said to be proper for a hypergraph $H = (V, E)$ if there is no monochromatic edges from $E$ under this coloring. The chromatic number of the hypergraph $H$, $\chi(H)$, is the minimum number of colors required for a proper coloring.

The talk deals with establishing the quantitative relationship between the chromatic number and the maximum edge degree for uniform hypergraphs from different classes. Starting with the classical paper of Erdős and Lovász [1] this type of questions was in the center of the hypergraph coloring theory, a several well-known probabilistic tools were developed during the study of them.

We consider the class of $b$-simple hypergraphs. Recall that a hypergraph is said to be $b$-simple if every two of its distinct edges do not share more than $b$ common vertices. In the paper [2] the authors proved that if $H$ is a $1$-simple $k$-uniform hypergraph with $\chi(H) > r$ then $\Delta(H) = \Omega(k \cdot r^{k-1})$. Our new result extends this statement to the case $b > 1$.

**Theorem.** Suppose $b \geq 1$, $r \geq 2$ and $k > k_0(b)$ is large enough in comparison with $b$. If a $b$-simple $k$-uniform hypergraph $H$ satisfies the inequality

$$\Delta(H) \leq c \cdot k r^{k-b},$$

(2.18)

where $c > 0$ is some absolute constant, then $H$ is $r$-colorable.

Theorem improves the previously known bound from [3]. For fixed $r, b$ and large $k$, the obtained estimate (2.18) is only $\Theta(k \ln r)$ times smaller than the best known upper bound from [4]. The proof relies on the application of the random recoloring method and a special case of the Lovász Local Lemma.

**References**


Estimates of the entropy and Kantorovich distances between stationary distributions of diffusions

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We estimate the total variation, entropy and Kantorovich distances between stationary distributions of two diffusion processes. This estimate is applied to nonlinear stationary Kolmogorov equations. Suppose that a diffusion process \( \xi_t \) with generator \( L_{A,b,u} = \text{tr}(AD^2u) + \langle b, \nabla u \rangle \) possesses an invariant probability measure \( \mu \). Then \( \mu \) satisfies the stationary Kolmogorov equation \( \sum_{i,j=1}^d \partial_x \partial_{x_j} \left(a_{ij} \mu\right) - \sum_{i=1}^d \partial_{x_i} \left(b_i \mu\right) = 0 \). The following results are given in [1].

**Theorem.** Assume that \( A = I \). Let \( \mu \) and \( \nu = v \cdot \mu \) be two probability solutions to the stationary Kolmogorov equations with locally bounded Borel coefficients \( b_\mu \) and \( b_\nu \), respectively. Suppose that \( |b_\mu - b_\nu| \in L^2(\nu) \) and that at least one of the following two conditions is fulfilled: \( 1 + |x|^{-1}|b_\mu(x)| \in L^1(\mu) \) or there exists a function \( V \in C^2(\mathbb{R}) \) such that \( L_{1,b_\nu}V(x) \leq MV(x) \) for all \( x \) and some \( M > 0 \) and \( \lim_{|x| \to \infty} V(x) = +\infty \). Then \( b_\mu - b_\nu, \nabla V(1 + V)^{-1} \in L^1(\nu) \). Then there holds the estimate

\[
\int_{\mathbb{R}^d} \frac{|\nabla v|^2}{v} \, dv \leq \int_{\mathbb{R}^d} |b_\mu - b_\nu|^2 \, dv.
\]

**Corollary.** Suppose that, in addition to the hypotheses of the theorem, it is assumed that the solutions \( \mu \) and \( \nu \) have second moments and the measure \( \mu \) satisfies the logarithmic Sobolev inequality with constant \( C \). Then

\[
W_2(\mu, \nu) \leq C \frac{\|b_\mu - b_\nu\|_{L^2(\nu)}}{\sqrt{C} \|b_\mu - b_\nu\|_{L^2(\nu)}},
\]

where \( W_2 \) is the quadratic Kantorovich metric and \( \| \cdot \|_{TV} \) is the total variation distance.

Analogous results for stationary distributions of two diffusion processes with different diffusion matrices are given in [2]. In particular, the following assertion holds.

**Theorem.** Let \( \mu \) and \( \nu \) be two probability solutions to the stationary Kolmogorov equations with locally bounded Borel coefficients \( b_\mu, a_{ij}^\mu \) and \( b_\nu, a_{ij}^\nu \). Let \( A_{\mu,\nu}(x) \geq \alpha I, |A_{\mu,\nu}(x) - A_{\mu,\nu}(y)| \leq \Lambda |x - y|, |b_\mu(x) - b_\nu(y), x - y| \leq -\kappa |x - y|^2 \) for some numbers \( \alpha > 0, \Lambda > 0 \) and \( \kappa > 0 \). Suppose that \( |b_\mu| \in L^1(\mu + \nu), |x| \in L^1(\nu) \) and \( |\Phi| \in L^1(\nu) \), where

\[
\Phi = \frac{(A_{\mu,\nu} - A_\mu)\nabla b_\nu}{b_\nu} - (h_\mu - h_\nu), \quad h_\mu^i = b_\mu - \partial_{x_j}(a_{ij}^\mu), \quad h_\nu^i = b_\nu - \partial_{x_j}(a_{ij}^\nu).
\]

If \( m = \kappa - (4\alpha)^{-1}d^2A^2 > 0 \), then \( W_1(\mu, \nu) \leq \frac{1}{m} \int_{\mathbb{R}^d} \Phi \, d\nu \) and \( \|\mu - \nu\|_{TV} \leq C(d, \alpha, \Lambda, \kappa) \int_{\mathbb{R}^d} |\Phi| \, d\nu \).

A survey of results about Fokker–Planck–Kolmogorov equations is presented in [3].

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The long term behaviour of two interacting birth-and-death processes

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This talk concerns the long term behaviour of stochastic models formulated in terms of two interacting birth-and-death processes. In absence of interaction the model is just a pair of two independent birth-and-death processes, whose long term behaviour is well known. Namely, given a set of transition rates one can, in principle, determine whether the corresponding birth-and-death process is recurrent/positive recurrent, or transient/explosive, and compute various characteristics of the process. Presence of interaction can significantly affect the collective behaviour. It should be noted that a birth-and-death process is a classic stochastic model for the size of the population, therefore interacting birth-and-death processes provide a natural mathematical framework for modelling interaction between populations. In this talk one of the models of interest is a stochastic population model for competition between two species with Lotke-Volterra type of interaction.

Recent advances in the estimation of the rate of convergence in the CLT for sums of independent random variables

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We consider the problem of estimation of the accuracy of the normal approximation to distributions of sums of independent random variables with finite second-order moments, or higher. We present natural convergence rate estimates in the classical Lindeberg-Feller theorem in terms of truncated moments improving Osipov, Katz-Petrov, Esseen, Rozovsky, and Ahmad-Wang inequalities, as well classical bounds of the Berry-Esseen-type.

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Ornstein-Uhlenbeck type growth-fragmentation processes

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Growth-fragmentation processes describe systems of particles in which each particle may grow larger or smaller, and divide into smaller ones as time proceeds. Unlike previous studies, which have focused mainly on the self-similar case, we introduce a new type of growth-fragmentation which is closely related to Lévy driven Ornstein-Uhlenbeck type processes. Our model can be viewed as a generalization of compensated fragmentation processes introduced by Bertoin (Ann. Probab. 2016). We establish a convergence criterion for a sequence of such growth-fragmentations. We also prove that, under certain conditions, the average size of the particles converges to a stationary distribution as time tends to infinity.
Scaling limits of Crump-Mode-Jagers trees

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Crump-Mode-Jagers (CMJ) branching processes generalize the classical Galton-Watson (GW) branching processes by allowing individuals to live for a duration \( V > 0 \) and give birth during their life time at random times described by a point measure \( \mathcal{P} \) supported on \((0, V] \). The genealogy of CMJ processes is given by chronological trees where edges have real-valued length given by \( V \) and are stuck at the atoms of the point measures \( \mathcal{P} \).

In the critical case \( E(\mathcal{P}(\mathbb{R}_+)) = 1 \), it is now well-known that the height and contour processes associated to GW processes converge to a continuous object called the height process and denoted by \( H_1 \). In the more general case of CMJ processes, much less is known and I will present in this talk recent results in this context. To state this result informally, let \( H_n \) and \( C_n \) be the scaled chronological height and contour processes associated to the random chronological trees coding CMJ processes. The following conditions involve the random variable \( Y \) which corresponds, informally, to the age of the parent of a typical individual, say \( u \), when giving birth to \( u \). The rigorous definition involves the weak ascending ladder height times and the undershoots of the Lukasiewicz path.

**Theorem.** If \( E(Y), E(V) < \infty \), then \( (H_n, C_n) \overset{fdd}{\to} (E(Y)(H_\infty, H_\infty(\cdot/E(2V)))) \)

In words: when edges are “short” (which is the informal meaning of the condition \( E(Y), E(V) < \infty \)), then the chronological height and contour processes coincide asymptotically with the discrete height and contour processes upon a stretching of the edges by the deterministic factor \( E(Y) \).

Our second main result concerns the case where the offspring distribution has finite variance. In this case, the limit of the chronological height process is expressed in terms of the \( \beta \)-stable Poisson snake \(((H_\infty(t), \Pi_t), t \in \mathbb{R}_+)\), which is the unique stochastic process with the following property: conditionally on \( H_\infty \), \( \Pi_t \) is a path-valued Markov process with the following transition mechanism: for every \( s \leq t \),

\[
\Pi_t \overset{df}{=} \Pi_{s \wedge [t-s,t]} H_\infty + \Theta_{-s \wedge [t-s,t]} H_\infty \left( \Gamma_{H_\infty(t)-s \wedge [t-s,t]} H_\infty \right)
\]

where \( \Gamma \) is an independent \( \beta \) stable subordinator. In the previous formula, \( f \mid_t \) is the function \( f \) stopped at \( t \) and \( \Theta_t(f) \) is the function \( f \) shifted at \( t \): \( f \mid_t(s) = f(s \wedge t) \) and \( \Theta_t(f)(s) = f(t+s) \).

**Theorem.** If \( \mathcal{P}(\mathbb{R}_+) \) has finite variance and \( Y \) is in the domain of attraction of a \( \beta \) stable distribution with \( \beta \in (0, 1) \), then \( H_n \overset{fdd}{\to} H_\infty \) where \( H_\infty \) is the terminal value of a Poisson snake driven by \( H_\infty \), i.e., \( H_\infty(t) = \Pi_t(H_\infty(t)) \).

In words: when the offspring distribution has finite variance, the chronological tree is asymptotically obtained by marking the genealogical tree in a Poissonian way.

**References**


Branching process approach for statistical estimation of cancer recurrence

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The main goal of this study is to consider multitype age-dependent branching processes (in continuous time) to model the dynamics of different types of organisms, which are fated to become extinct, due to a small reproductive ratio $R < 1$, which means that one organism could produce on average less than one offspring per lifetime. However, mutations occurring during the reproduction process, may lead to the appearance of new types of organisms with $R > 1$, that may escape extinction. This is a common structure observed when cancer cells are escaping from chemotherapy, as well as, when viruses or microbes are evading anti-microbial therapy. This research is a generalization of the two-type age-dependent branching model developed in the recent work [1]. We are deriving mathematically the number of different types of mutations leading to the escape type and their moments. There are several differences from the results obtained in [2] where a discrete-time branching processes were used. One of them is that the behaviour of the probability of mutations appearance, provided it is not happened yet, depends essentially on the lifetime distribution of the cells.

More precisely, the appearance of mutations in cancer development plays a crucial role in the disease control and its medical treatment. Motivated by the practical significance, it is of interest to model the event of occurrence of a mutant cell that will possibly lead to a path of indefinite survival. A multi-type branching process model in continuous time is proposed for describing the relationship between the waiting time till the first escaping extinction mutant cell is born and the lifespan distribution of cells, which due to the applied treatment have small reproductive ratio $R < 1$. All the studied quantities throw light on the estimation of the risk of recurrence, which depending on the context might be such as of cancer recurrence or anti-biotic resistance, etc.

A numerical method and related algorithm for solving the integral equations is developed, in order to estimate the distribution of the waiting time to the escaping extinction mutant cell is born. Numerical studies demonstrate that the proposed approximation algorithm reveals the substantial difference of the results in discrete-time setting. In addition, to study the time needed for the mutant cell population to reach high levels a simulation algorithm for continuous multi-type decomposable branching process is proposed. Two different computational approaches together with the theoretical studies might be applied to different kinds of cancer and their proper treatment.

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Markov chain tree theorem and related problems

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There is a remarkable theorem in the theory of finite Markov Chains (MCs) - MC Tree theorem (MCTT), equivalent to a theorem initially discovered by G. Kirchhoff, expressing the limiting distribution $\pi$ for an ergodic MC in terms of directed spanning trees. This theorem serves as a "bridge" between MCs and Graph Theory. The values of $\pi(y)$ are the normalized values of $q(y)$, where $q(y)$ are obtained as follows. For every spanning tree directed to point $y$ one has to calculate the product of transition probabilities over all edges of this tree, and $q(y)$ is the sum over all trees directed to $y$. The applications of MCTT are limited by the fact that the number of trees directed to a point growing exponentially. In paper (Sonin, 1999) it was noted and proved that there is a polynomial algorithm to calculate $q(y)$ having a simple probabilistic interpretation. The proof was complicated and used some subtle facts from the graph theory. This algorithm and corresponding theorem were generalized into the case of so called idempotent (tropical) calculus in paper (Gursoy et al., 2015). Another proof of the same theorem was given in (Fomin et al., 2016). A new proof of this theorem without using any results from the graph theory will be outlined.

References


From optimal transport to Ricci flow — gradient flows, heat equation, and Brownian motion on time-dependent metric measure spaces.

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We study the heat equation on time-dependent metric measure spaces (being a dynamic forward gradient flow for the energy) and its dual (being a dynamic backward gradient flow for the Boltzmann entropy). Monotonicity estimates for transportation distances and for squared gradients will be shown to be equivalent to the so-called dynamical convexity of the Boltzmann entropy on the Wasserstein space. The latter is a robust characterization of so-called super-Ricci flows of metric measure spaces.
Analysis of Stratified Mark-Specific Proportional Hazards Models under Two-Phase Sampling with Application to HIV Vaccine Efficacy Trials

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An objective of preventive HIV vaccine efficacy trials is to understand how immune responses to specific protein or sub-protein sequences of HIV associate with the level of vaccine efficacy to prevent infection with sequences of HIV targeted by the immune responses. The vaccine-induced immune response biomarkers are often measured via two-phase sampling for efficiency. Motivated by this objective, we investigate the stratified mark-specific proportional hazards model under two-phase sampling, where the mark is the genetic distance of an infecting HIV sequence to an HIV sequence represented inside the vaccine. The estimation and inference procedures based on inverse probability weighting of complete-cases and the augmented inverse probability weighted complete-case are developed. The asymptotic properties are derived, and their finite-sample performances are examined in a comprehensive simulation study. The methods are shown to have satisfactory performance, and are applied to the RV144 vaccine trial to assess whether immune response correlates of HIV infection are stronger for HIV infecting sequences similar to the vaccine than for sequences distant from the vaccine.

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Heavy-tailed fractional Pearson diffusions

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Recent developments in fractional processes, including fractional diffusions, have been motivated by applications that require modeling of various phenomena using fractional partial differential equations. In this talk we focus on one particular fractional diffusion, obtained by a specific time-change in the diffusion process with invariant Fisher-Snedecor distribution which belongs to the class of the so-called heavy-tailed Pearson diffusions, see e.g. Avram et al. (2013a).
Pearson diffusions are a special class of diffusion processes governed by the Kolmogorov backward equation (KBE) with varying polynomial coefficients: the drift coefficient is polynomial of the first degree and the diffusion coefficient is polynomial of at most second degree. The study of classical Markovian Pearson diffusions began with Kolmogorov in 1930’s and then continued by Wong in 1964. Analysis and applications of spectral properties of these diffusions are treated in the recent paper by Avram et al. (2013b). Diffusion processes from this family include the famous Ornstein-Uhlenbeck (OU) process and the Cox-Ingersoll-Ross (CIR) process that are widely used in applications. Other Pearson diffusions are Jacobi diffusion with beta invariant distribution, reciprocal gamma, Fisher-Snedecor and Student diffusions, named after their invariant distributions. The last three are known as heavy-tailed Pearson diffusions.

Pearson diffusions can be defined as stochastic processes by specifying their Markovian nature, their transition density via the corresponding KBE and the distribution of the initial value. Fractional diffusions, due to the non-Markovian nature, generally cannot be defined as stochastic processes by the governing equations and the initial distributions alone, and therefore for their definition alternative approaches are required. For the three non-heavy-tailed Pearson diffusions (OU, CIR, Jacobi), their fractional analogs were defined in Leonenko et al. (2013).

In this talk we extend the results from Leonenko et al. (2013) to the fractional Fisher-Snedecor diffusion (FSD). We define this fractional diffusion by time changing the corresponding non-fractional heavy-tailed diffusion by the inverse of the standard stable subordinator. Next, we obtain the spectral representations for its transition densities and describe the first-order and second-order properties of this process. Finally, we provide the explicit strong solutions to the KBE for fractional FSD using properties of the spectrum of its generator with polynomial coefficients. More details on results presented in this talk could be found in the recent paper by Leonenko et al. (2017).

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Uniqueness of dirichlet forms related to infinite systems of interacting brownian motions

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We study Dirichlet forms related to infinite systems of interacting Brownian motions. Introducing the bilinear form defined on the set of local smooth functions on the configuration space, we consider two kinds of extensions of it. These are related to the infinite system obtained by the limits of finite systems of interacting Brownian motions with boundary conditions of absorbing type and of reflecting type, respectively. We give sufficient conditions for the coincidence of them.

Acknowledgement: This talk is based on the joint work with Yusuke Kawamoto and Hirofumi Osada.
Invariance of closed convex cones for stochastic partial differential equations

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The goal of this paper is to clarify when a closed convex cone is invariant for a stochastic partial differential equation (SPDE) driven by a Wiener process and a Poisson random measure, and to provide conditions on the parameters of the SPDE, which are necessary and sufficient.

References


Fractality of wave functions on a Cayley tree

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The problem of Anderson localization on hierarchical lattices has attracted attention recently in the context of many-body localization problem. We investigate analytically and numerically eigenfunction statistics in a disordered system on a finite Bethe lattice and Random Regular Graphs. For Bethe lattice, we show that the wave function amplitude at the root of a tree is distributed fractally in a large part of the delocalized phase [1]. The fractal exponents are expressed in terms of the decay rate and the velocity in a problem of propagation of a front between unstable and stable phases. We demonstrate a crucial difference between a loopless Cayley tree and a locally tree-like structure without a boundary (random regular graph) where extended wavefunctions are ergodic. For RRG, we focus on the delocalized side of the transition and stress the importance of finite-size effects. We show that presence of even very large loops (with size comparable with the diameter of the structure) modifies the properties of delocalized phase a lot. We show that the numerics can be interpreted in terms of the finite-size crossover from small to large system, separated by the correlation volume, diverging exponentially at the localization transition. A distinct feature of this crossover is a nonmonotonicity of the spectral and wavefunction statistics, which is related to properties of the critical phase in the studied model and renders the finite-size analysis non-trivial [2].

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References


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Let $Z'(t) = (Z'_1(t), Z'_2(t), \ldots, Z'_n(t))$, $t \geq 0$, be an indecomposable, aperiodic and critical $n$-type Bellman-Harris branching process initiated at time zero by a single particle of type $i \in \{1, 2, \ldots, n\}$. A type $i$ particle of the process has life-length $\tau_i > 0$ with distribution $G_i(t)$ and produces at the end of its life a random number of children of different types specified by the vector $(\xi_{i1}, \xi_{i2}, \ldots, \xi_{in})$ which meets the condition: $E\xi_{ij} < \infty$ for all $i, j, s$.

Set $G_i(t) = 1 - q_i(t)$ and $\mu_j := E\tau_j \leq \infty$. Suppose that there exists $0 \geq n_0 < n$ such that

- if $j > n_0 + 1$ then $\mu_j = \infty$ and $q_j(t) = 1 - G_j(t) = t^{-\beta_j} \ell_j(t)$,

where $\beta_j \in (0, 1]$, $\ell_j(t)$ is a function slowly varying at infinity and $q_j(t) = O(q_n(t))$;

- if $j \leq n_0$ then $\mu_j < \infty$ and, additionally, $q_j(t) = o(q_n(t))$ if $\beta_n = 1$.

Let

$$
\mu_j(t) = \int_0^t q_j(u)du, \quad \nu_j(t) := \mu_j(t)/\mu_n(t).
$$

Denote by $A_j$, the condition $\int_0^\infty \nu_j^2(t)dt < \infty$.

If all the conditions above are valid and, in addition, if $\beta_n \in (0, 0.5]$ then there exists a constant $C > 0$ such that

$$
G_i(t + \Delta) - G_i(t) \leq C t^{-1 - \beta_n} \ell_n(t)
$$

for all fixed $\Delta > 0$, $t > 1$ and all $i$, then

$$
EZ_i^j(t) \sim c_{ij} \nu_j(t), \quad t \to \infty,
$$

(2.19) for explicitly known constants $c_{ij} > 0$.

If the conditions providing (1) are valid and, in addition, if $\beta_n \in (0.5, 1]$ then

$$
G_i(t + \Delta) - G_i(t) = o(t^{-1}), \quad t \to \infty,
$$

for all fixed $\Delta > 0$ and all $i$, then

$$
EZ_j^i(t)^2 \sim c_{ij}^{(l)} \mu_n^{-1}(t) \int_0^t \nu_j(u)du + 1_{(A_j^l)} c_{ij}^{(2)} \nu_j(t), \quad t \to \infty,
$$

for explicitly known constants $c_{ij}^{(l)} > 0$, $l = 0, 1$.

Some of the statements above for the case $n = 2, n_0 = 1$ are contained in [1] and [2].

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Numerical methods in branching processes and applications

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In branching process theory often arise renewal-like integral equations we need to solve. Such equations are used to describe a particular distribution of a random variable we are interested in, the expected population count or a probability generating function. Although they are similar to the renewal equations, for which we have an explicit form of the solution as a convolution of two functions, we cannot apply renewal theory to solve them. Examples can be found almost everywhere - Sevastyanov branching processes (see [1,2]), in general branching processes (see [3,4]), cancer growth with multi-type Bellman-Harris branching process with mutation (see [5]) and many others. A theoretical solution is particularly difficult in the general case and only some solutions exist for special cases, e.g. when the integral equation is renewal and the time between renewals is exponentially distributed. Often in practice however we require the use of non-parametric distributions in order to model more accurately the observed data, which requires using a numerical approach for solving the related integral equations. This paper presents a numerical method for solving the required integral equations, where only smoothness conditions are imposed, which allows the model to be tailored more accurately to the observed empirical data and proves the numerical estimation error to be of order \( O(h) \). Finally, some applications are presented for different cases of branching processes.

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References


Where do we get using random walk theory for flow analysis?

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I will address the question of interrelation between the dynamics of the continuous system and the topology of the discrete object (such as a graph), which, to some extent, characterizes this continuous system dynamics. For this I used the framework of flow-networks, which is applicable to analyse Lagrangian and Eulerian static flow systems. I extended the flow-networks framework for analysis of broader class of dynamical non-equilibrium, nonautonomous
systems. What can be learned from the general flow-networks method and, importantly, what are the method limitations? The general flow-networks method showed the new insights about the system dynamics: the correlation network measures for non-autonomous systems strongly depend on relative scales of advective and dissipative components of the dynamical system, surprisingly, the dissipation creates "memory" effects even in markovian systems, and, moreover, the time-dependent forcing does not affect the correlation network topology.

Windings of planar stochastic processes

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Two-dimensional (planar) processes attract the interest of many researchers. This happens both because of their richness from a theoretical point of view and because their study turns out to be very fruitful in terms of applications (e.g. in Finance [9], in Biology [8] etc.). This talk focuses on the fine study of trajectories of planar processes, and in particular on their windings.

We will survey several results concerning windings of two-dimensional processes, including planar Brownian motion (BM), complex-valued Ornstein-Uhlenbeck (OU) processes and planar stable processes (see e.g. [3,7]). We will also present Spitzer’s asymptotic Theorem for each case.

Our starting point will be the skew-product representation. Then, we will introduce Bougerol’s celebrated identity in law [4] which is very useful for the study of the windings of planar BM and of complex-valued OU processes, stating that for $u > 0$ fixed,

$$\sinh(\beta_u) \overset{(law)}{=} \beta_{A_u(\beta)} = \int_0^{\beta_u} ds \exp(2s)$$

where $(\beta_t, t \geq 0)$ and $(\tilde{\beta}_t, t \geq 0)$ are two independent linear Brownian motions, and the second one is also independent from $A_u(\beta)$. However, this method cannot be applied for the case of planar stable processes [1]. So, we will tackle this problem firstly by using new methods invoking the continuity of the composition function [5] and secondly by applying new techniques from the theory of self-similar Markov processes [6] having as a starting point the so-called Riesz–Bogdan–Zak transform introduced in [2] which gives the law of the stable process when passed through the spatial Kelvin transform and an additional time change.

Acknowledgement: I am indebted to Professors Ron Doney, David Holcman, Andreas Kyprianou and Marc Yor (who, unfortunately, passed away unexpectedly recently) with the collaboration of whom several of the above topics where developed. I would also like to thank the University of Cyprus (Project: Postdoctoral Researchers 2016-2017) and the Organizing Committee of the SPA2017 who both finance partially my participation to the 39th Conference on Stochastic Processes and their Applications.

References

Averaged control for 1D ergodic diffusions

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An ergodic HJB (Bellman) equation is proved for a 1D controlled stochastic differential equation (SDE)

$$dX_t^a = b(\alpha(X_t^a), X_t^a) \, dt + \sigma(\alpha(X_t^a), X_t^a) \, dW_t, \quad t \geq 0, \quad X_0^a = x,$$

with variable diffusion and drift coefficients both depending on the control strategy $\alpha$, with a running cost function $f$ and a price

$$\rho^\alpha := \limsup_{T \to \infty} \frac{1}{T} \int_0^T \mathbb{E}_x f(\alpha(X_t^a), X_t^a) \, dt \to \min_{\alpha}.$$

Here $\alpha = \alpha(x), \, x \in R^1$, is a feedback strategy which is a Borel function with values from a compact set. Beside the HJB equation, the second goal of this work is to find a way to compute an optimal or close to optimal strategy $\alpha$.

The diffusion – a weak solution of the SDE above – is assumed uniformly stable which is guaranteed by suitable conditions on the coefficients. All coefficients and the function $f$ are assumed bounded and the diffusion $\sigma^2$ uniformly non-degenerate; a certain additional smoothness of all coefficients including $f$ with respect to $x$ is required; all coefficients are assumed to be continuous in the first variable $a$. The ergodic Bellman equation has the form,

$$\inf_{\alpha} \left\{ \frac{1}{2} \sigma^2(a, x)v''_{xx}(x) + b(a, x)v'_x(x) + f(a, x) + \rho \right\} = 0,$$

where $v(x)$ is an auxiliary function. It is proved that a solution $(v(\cdot), \rho)$ of the latter equation exists and that it is unique in the following class of pairs: $v$ is a polynomially growing function with two derivatives, $\rho$ is a constant; of course, for the component $v$ uniqueness is understood up to an additive constant and this additive constant is of no importance for finding $\rho$: indeed, note that only derivatives of $v$ are present in the HJB equation. It is also proved that the unique in a usual sense second component $\rho$ equals the desired price $\inf_{\alpha} \rho^\alpha$. Convergence of the iterative price improvement algorithm to $\rho$ is established. In a slightly different setting – in particular, we only treat “strong” strategies and do not touch “weak” or relaxed ones – our results in a certain (not fully specified) sense extend to the case of controlled $\sigma$ the results from [1, Chapter 3] in a “stable case”. Certain ideas and techniques from [2, Chapter 1] have been used.

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References

Independence times for iid sequences, random walks and Lévy processes

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It is well-known that for a sequence with independent identically distributed (iid) values (respectively, for a process with stationary independent increments, i.e. for a random walk or Lévy process) $X$ and a random time $R$ of $X$, the strict post-$R$ sequence (respectively, the process of the increments of $X$ after $R$) (i) is independent of the history up to $R$ and (ii) has the same distribution as $X$, provided $R$ is a stopping time of $X$. It is less well-known, and somewhat remarkable, that properties (i) and (ii) already force $R$ to be a stopping time of $X$ (once the notion of the history up to $R$ has been given a precise meaning). In the talk we consider those random times $R$ of $X$ for which (i) obtains (not all of which are, in general, stopping times of $X$). Referring to them briefly as independence times, then for random walks/iid sequences we present their set-theoretic characterization, whilst for Lévy processes reasonably useful sufficient conditions and a partial necessary condition for independence times are given. The results are closely related to the study of conditional independence/Markov times from the general theory of Markov processes.

Asymptotic behaviour of interacting birth-and-death processes

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We study locally-interacting birth-and-death processes on nodes of a finite connected graph; the model which is motivated by modelling interactions between populations, adsorption-desorption processes, and is related to interacting particle systems, Gibbs models, and interactive urn models. Alongside with general results, we obtain a more detailed description of the asymptotic behaviour in the case of certain special graphs.

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Exponential functionals of PII and their applications in mathematical finance

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Exponential functionals arise in many areas, in particular in the theory of self-similar Markov processes in the relation with Lamperti transform, in the theory of random processes in random environments, in the area of mathematical statistics, for example, in the study of Pitman estimators, in the mathematical finance in relation with the perpetuities containing the liabilities, with the perpetuities subjected to the influence of the economical factors, with the prices of the Asian options and also with the ruin problem of the insurance companies.

In the first part of this talk we consider the exponential functionals

$$I_t = \int_0^t \exp(-X_s) ds, \ t \geq 0,$$

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and also

\[ I_\infty = \int_0^\infty \exp(-X_s)ds, \]

with the process \( X \) being a process with independent increments (PII) and also a semi-martingale with the absolutely continuous characteristics. We give a recurrent integral equation for Mellin transform \( E(I_t), t \in \mathbb{R}, \) of the exponential functional \( I_t \). We deduce from this result the formulas for the moments of exponential functionals. In the case of Levy processes this gives us an explicit answer for the moments of \( I_t \) and \( I_\infty \), under less restrictive conditions than in Bertoin, Yor (2005).

The second part of this talk is devoted to the distributions of exponential functionals of the processes with independent increments. We give an integro-differential equations for the density of the integral functional \( I_t \), when this density exists and belong to the class \( C^{1,2}_b(\mathbb{R}^+, \mathbb{R}) \). In the particular case of Levy processes, the equation for the density can be simplified and can be solved in a number of cases. We mention here the case of Brownian motion with drift studied by Dufresne (1990). From the previous result we deduce, as a particular case, the equations for the density of \( I_\infty \) which are similar to the equations given in Carmona, Petit, Yor (2004).

In the third part of this talk we consider ruin problem for insurance companies. More precisely we will consider Cramér-Lundberg model with investment involving Levy processes. We show why the behaviour of the exponential functional of the investment part determine the asymptotic for the ruin probability, and explain the exponential decay, polynomial decay of the ruin probability, and the ruin with the probability 1.

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**Stability of overshoots of zero mean random walks**

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Take a one-dimensional random walk with zero mean increments, and consider the sizes of its overshoots over the zero level. It turns out that this sequence, which forms a Markov chain, always has a stationary distribution of a simple explicit form. The questions of uniqueness of this stationary distribution and convergence towards it are surprisingly hard. We were able to prove only the total variation convergence, which holds for lattice random walks and for spread out random walks (the ones whose distribution has a vanishing singular component as the number of steps increases). In addition, we obtained the rate of this convergence under additional mild assumptions. We will also discuss connections to related topics: local times of random walks at the zero level, stability of reflected and oscillating random walks, ergodic theory, and renewal theory. This is a joint work with Alex Mijatovic (King’s College London).

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**Persistence of integrated random walks**

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In this talk I’ll discuss the asymptotic behaviour of the probability that the area under a centered random walk remains positive up to a large time \( n \). This problem has attracted a lot of attention in the recent past and there are at least three quite different methods, which allow one to derive asymptotic properties of the persistence probability under different restrictions on the increments of the random walk. I’ll describe all these results and the differences between existing approaches.
Local Stability of the Resolvent Flow under Dyson Brownian Motion 
and the Phase Transition in the Ultrametric Ensemble

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Simone WARZEL  
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In this talk we study the flow of the Green functions of $N \times N$ stochastic matrices with a random diagonal component under Dyson Brownian motion. The main theorem concerns optimal stability results up to times of order $N^{1/2}$ when the complex energy parameter is of the order $N^{-1}$. These results thus complement existing proofs of equilibration of the local statistics for times beyond $N^{-1}$. We also present two main applications: the Rosenzweig-Porter random matrix model as well as the ultrametric ensemble. In both cases, we map out the entire localized phase of in terms of both eigenfunctions and local statistics. Applying existing results for the delocalized phase, we thus establish a phase transition in those ensembles.

Acknowledgement: This work was supported by the DFG (WA 1699/2-1).

Multiplicative chaos in number theory and random matrix theory

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I will discuss how certain random measures and random generalized functions known as multiplicative chaos measures and distributions arise naturally when considering the statistical behavior of the Riemann zeta function on the critical line as well as when considering characteristic polynomials of suitable random matrices.

Aspects of the seed bank vs. the two-island model

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Population Genetics is known to be an area of probability theory where interesting math is driven by application in biology. Aiming to describe the evolution of a population with active and dormant forms (‘seeds’), we developed a model Markovian in both directions of time, whose limiting objects posses the advantageous property of being moment duals of each other: The (biallelic) Wright Fisher diffusion with seed bank component describing the fraction of a specific type of alleles forward in time and a new coalescent structure named seed bank coalescent describing the genealogy backward in time, [2].

In this talk we will give a brief introduction to these structures (extended to include mutation, [1]) comparing it to the structurally similar Two Island model (studied for example in [4]), with an emphasis on the evolution forward in time. This leads us to the study of a two-dimensional diffusion encompassing both models. We focus on its boundary behaviour making use of polynomial diffusion theory, cf. [3], by means of which we outline similarities and differences between the two models.

Based on joint work with J. Blath (TU Berlin), E. Buzzoni (TU Berlin), B. Eldon (Museum für Naturkunde Berlin), A. González Casanova (WIAS Berlin), N. Kurt (TU Berlin).
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**Hypergeometric SLE and Convergence of Critical Planar Ising Interface**

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Conformal invariance and critical phenomena in two-dimensional statistical physics have been active areas of research in the last few decades. This talk concerns conformally invariant random curves that should describe scaling limits of interfaces in critical lattice models.

The scaling limit of the interface in critical planar lattice model with Dobrushin boundary conditions (b.c.), if exists, should satisfy conformal invariance (CI) and domain Markov property (DMP). In 1999, O. Schramm introduced SLE process, and this is the only one-parameter family of random curves with CI and DMP. In 2010, D. Chelkak and S. Smirnov proved that the interface of critical Ising model on the square lattice does converge to SLE(3). In this talk, we discuss the scaling limit of the pair of interfaces in rectangle with alternating b.c. The scaling limit of the pair of interfaces, if exists, should satisfy CI, DMP and symmetry (SYM). It turns out there is a two-parameter family of random curves satisfying CI, DMP, and SYM, and they are Hypergeometric SLE. For the critical Ising model on the square lattice, the pair of interfaces does converge to Hypergeometric SLE(3). In this talk, we will explain two different proofs for the convergence. Furthermore, we will discuss results about global and local multiple SLEs, which correspond to the scaling limit of the collection of interfaces with alternating b.c. in more general setting.

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References


Normal approximation for statistics of Gibbsian input in geometric probability

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We consider the asymptotic behaviour of a random variable $W_\lambda$ resulting from the summation of the functionals of a Gibbsian spatial point process over windows $Q_\lambda \to \mathbb{R}^d$, where $Q_\lambda$ is a window with volume $\lambda$. We establish conditions ensuring that $W_\lambda$ has volume order fluctuations, i.e. they coincide with the fluctuations of functionals of Poisson spatial point processes. We combine this result with Stein’s method to deduce rates of a normal approximation for $W_\lambda$ as $\lambda \to \infty$. Our general results establish variance asymptotics and central limit theorems for statistics of random geometric and related Euclidean graphs on Gibbsian input. We also establish a similar limit theory for claim sizes of insurance models with Gibbsian input, the number of maximal points of a Gibbsian sample, and the size of spatial birth-growth models with Gibbsian input. This is a joint work with J. E. Yukich.

Acknowledgement: ARC Discovery Grants DP130101123 and DP150101459.

On the equivalence between some jumping sdes with rough coefficients and some non-local PDEs

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We study some jumping SDE and the corresponding Fokker-Planck (or Kolmogorov forward) equation, which is a non-local PDE. We assume only some measurability and growth conditions on the coefficients. We prove that for any weak solution $(f_t)_{t \in [0,T]}$ of the PDE, there exists a weak solution to the SDE of which the time marginals are given by $(f_t)_{t \in [0,T]}$. As a corollary, we deduce that for any given initial condition, existence for the PDE is equivalent to weak existence for the SDE and uniqueness in law for the SDE implies uniqueness for the PDE. This extends some ideas of Figalli [1] concerning continuous SDEs and local PDEs.

References

Forefather distribution in Galton-Watson Processes with age
dependant structure in population

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In this paper we examine the structure of a variant of the Galton-Watson branching process with particular
reference to the distribution of the number of forefathers of the individuals in the current generation. Forefathers
in our case have been defined as all the individuals since zeroth generation who have contributed to the birth of
the individual under consideration. Starting with a simplified model with the offspring distribution being Poisson
but allowing the number of time periods that an individual can survive to vary within a limited range, we find the
distribution of the number of forefathers using extensive simulation. For the cases where an individual can survive
for 2 or 3 time periods, exact expression for expected number of individuals having “k” forefathers has been derived.
A similar simulation analysis is carried out assuming the offspring distribution to be binomial and also negative
binomial. Some interesting insights and possible applications are discussed. In addition, results on asymptotics have
been derived and verified using simulation.

Comparing parametric and non-parametric methods in Forecasting
Time Series

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The Box-Jenkins model is applied as a parametric method for time series analysis and fitting seasonal and non-
seasonal autoregressive moving average (SARIMA) models. But this procedure is not useful for short length and
nonlinear time series data. To overcome these problems, two nonparametric methods i.e. Artificial Neural Network
and Singular Spectrum Analysis are introduced.

The artificial neural networks (ANN) are general, flexible, nonlinear tools capable of approximating any sort of
arbitrary function. Due to their flexibility as function approximates, ANN are robust methods in tasks related with
pattern classification, the estimate of continuous variables and time series forecasting. In this latter case, ANN offer
several potential advantages with respect to alternative methods mainly SARIMA time series models when it comes
to dealing with problems concerning nonlinear data which do not follow a normal distribution.

Singular spectrum analysis (SSA) is a technique of time series analysis and forecasting. It combines elements of
classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing.
SSA aims at decomposing the original series into a sum of a small number of interpretable components such as a slowly
varying trend, oscillatory components and a structureless noise. It is based on the singular value decomposition (SVD)
of a specific matrix constructed upon the time series. Neither a parametric model nor stationarity-type conditions
have to be assumed for the time series. This makes SSA a model-free method and hence enables SSA to have a very
wide range of applicability.

In this article, after introducing the above methods, using simulation studies the effectiveness of these methods for
Short-term and long-term predictions are evaluated. The results show the Superiority of Singular Spectrum Analysis
compared to the other two methods in terms of the root mean square error of forecasting. Then application of these
methods and their accuracy in forecasting some real data are compared.

References


$L^q$-valued Burkholder-Rosenthal inequalities and sharp estimates for stochastic integrals

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In this talk we present the Burkholder-Rosenthal inequalities for $L^q$-valued martingales. This type of inequalities connects the $L^p$-norm of a martingale with a norm that depends on the conditional moments of the martingale differences. We use these inequalities to obtain Itô isomorphisms for $L^q$-valued stochastic integrals with respect to a random measure belonging to a broad class of compensated random measures, that for example contains compensated Poisson random measures. The Burkholder-Rosenthal inequalities in fact also provide us with Itô isomorphisms for $L^q$-valued stochastic integrals with respect to a general (Hilbert space-valued) martingale.

This talk is based on joint work with Sjoerd Dirksen.

Stochastic Evolution of Particle Systems with their Generation and Transport on Multidimensional Lattices

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For the study of stochastic evolution of particle systems on a noncompact phase space we apply an approach focused on continuous-time branching random walks on multidimensional lattices with a finite set of branching sources, see details in [1]. The main object of study is the limit distribution of particles on the lattice. Special attention is paid to branching random walks with large deviations [2]. The limit theorems on asymptotic behavior of the Green function for transition probabilities were established for random walks with both a finite and infinite variance of jumps [3]. The obtained results allow to study the front of branching random walk and the structure of the particle population inside of the front and near to its boundary. For supercritical branching random walks, it
is shown that the amount of positive eigenvalues of the evolutionary operator, counting their multiplicity, does not exceed the amount of branching sources on the lattice, while the maximal of these eigenvalues is always simple [4]. We demonstrate that the appearance of multiple lower eigenvalues in the spectrum of the evolutionary operator can be caused by a kind of ‘symmetry’ in the spatial configuration of branching sources. The presented results are based on Green’s function representation of transition probabilities of an underlying random walk and cover not only the case of the finite variance of jumps but also a less studied case of infinite variance of jumps.

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References


Averaged autoregression quantiles

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We consider quantile autoregression model. In the paper properties of averaged autoregression quantile were studied. The result shows that averaged autoregression quantile in this model is monotone in (0, 1), and is asymptotically equivalent to the quantile of the location model. Earlier similar results were obtained by [3] for linear regression models.

References

Analysis of Survival Data with Measurement Error

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Covariate measurement error has attracted extensive interest in survival analysis. Since Prentice (1982), a large number of inference methods have been developed to handle error-contaminated data, and most methods are addressed to proportional hazards models. In contrast to proportional hazards models, additive hazards models offer a flexible alternative to delineate survival data. However, there is relatively less research on measurement error effects under such models, although some authors investigated this problem. In this talk, I will discuss several methods to correct for measurement error effects under additive hazards models. These methods will be justified both theoretically and empirically.

Spatially inhomogeneous contact models and nonlocal Shrödinger operators

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The asymptotic behavior of stochastic infinite-particle systems in continuum can be studied in terms of evolution equations for correlation functions. For the stochastic contact model in the continuum the evolution equation for the first correlation function (i.e., the density of the system) is closed and it can be considered separately from equations for higher-order correlation functions. In this case we have the following evolution problem for $u \in C([0,\infty);\mathcal{E})$ associated with a nonlocal diffusion generator $L$:

$$\frac{\partial u}{\partial t} = Lu, \quad u = u(t, x), \quad x \in \mathbb{R}^d, t \geq 0, \quad u(0, x) = u_0(x) \geq 0.$$ 

in a proper functional space $\mathcal{E}$. As $\mathcal{E}$, we consider two spaces: $C_b(\mathbb{R}^d)$, the Banach space of bounded continuous functions on $\mathbb{R}^d$, and $L^2(\mathbb{R}^d)$. These spaces are corresponding to two different regimes in the contact model: systems with bounded density and ones essentially localized in the space.

The operator $L$ has the following form:

$$Lu(x) = -m(x)u(x) + \int_{\mathbb{R}^d} a(x-y)u(y)dy,$$

where $a(x) \geq 0$, $a(-x) = a(x)$, $a \in C_b(\mathbb{R}^d) \cap L^1(\mathbb{R}^d)$ is an even continuous bounded function such that $\int_{\mathbb{R}^d} a(x)dx = 1$. The function $a(x-y)$ is the dispersal kernel associated with birth rates in the contact model. The function $m(x)$ is related with mortality rates. We assume here that

$$m(x) \in C_b(\mathbb{R}^d), \quad 0 \leq m(x) \leq 1, \quad m(x) \to 1, \quad |x| \to \infty.$$ 

We are interesting in local perturbations of the stationary regime, when $m(x)$ is an inhomogeneous in space non-negative function. We prove that local fluctuations of the mortality with respect to the critical value $m(x) \equiv 1$ can push the system away from the stationary regime. As a result of such local perturbations, we will observe exponentially increasing density of population everywhere in the space. The goal of the work is to obtain conditions
on the mortality rates which give the existence of a positive discrete spectrum of the operator \( L \), and to prove the existence and uniqueness of a positive eigenfunction \( \psi(x) > 0 \) corresponding to the maximal eigenvalue \( \lambda_0 > 0 \). We call this function the ground state of the operator \( L \).

We prove that the ground state appears in two cases: either there exists such a region of any (small) positive volume, where the fluctuation \( V(x) \) is equal to 1, or \( V(x) \) is positive and less than 1 in a large enough region. We stress that the function \( V(x) \) should be bounded from above by 1, since the mortality \( m(x) \geq 0 \) is a non-negative function. Thus we observe for the nonlocal operator \( L \) new effects different from those of the Shrödinger operators.

We found that even in the high dimensional case some small (in the integral sense) perturbations \( V(x) \) of the mortality \( m(x) \) from the critical value \( m(x) \equiv 1 \) imply existence of the positive eigenvalue and the corresponding positive eigenfunction.

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**Stochastic Heat Equations with Values in a Riemannian Manifold**

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I will talk about the existence of martingale solutions to the stochastic heat equation (SHE) in a Riemannian manifold by using suitable Dirichlet forms on the corresponding path and loop space. Moreover, we present some characterizations of the lower bound of the Ricci curvature by functional inequalities of various associated Dirichlet forms.

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**Monotone convergence of the number of \( P_3 \) in \( G(n, p) \)**

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Let \( k, b \) be positive integers, \( b \leq k \). Let \( \xi_{b,k} \sim \text{bin}(k, b/k) \), \( \eta_b \sim \text{pois}(b) \). Moreover, set

\[
p_{b,k} = P(\xi_{b,k} < b) = \sum_{i=0}^{b-1} \binom{k}{i} \left( \frac{b}{k} \right)^i \left( 1 - \frac{b}{k} \right)^{k-i}, \quad p_b = P(\eta_b < b) = \sum_{i=0}^{b-1} \frac{\lambda^i e^{-\lambda}}{i!}.\]

The number \( b \) is the median of \( \text{bin}(k, b/k) \) and \( \text{pois}(b) \). By Poisson limit theorem, \( p_{b,k} \to p_b \) as \( k \to \infty \). The question is, how close are the probabilities \( p_{b,k}, p_b \) to 1/2?

**Theorem** (Ramanujan, 1911; Szegö, 1928; Watson, 1929). Define a sequence \( y_b \) in the following way:

\[
\frac{1}{2} = p_b + y_b P(\eta_b = b).
\]

Then, for all \( b \), \( \frac{1}{3} < y_b \leq \frac{1}{2} \) and \( y_b \) decreases to \( \frac{1}{3} \) as \( b \to \infty \).

**Theorem** (K. Jogdeo, S. M. Samuels, 1968). For any positive integer \( b \), define a sequence \( z_{b,k} \), \( k \in \mathbb{N}, k > 2b \), in the following way:

\[
\frac{1}{2} = p_{b,k} + z_{b,k} P(\xi_{b,k} = b).
\]

Then, for every \( b \), \( z_{b,k} \) decreases for \( k \geq 2b \) and \( z_{b,k} \to y_b \) as \( k \to \infty \). Moreover, for all \( k > 2b \), \( \frac{1}{3} < z_{b,k} < \frac{1}{2} \). For all \( b < k < 2b \), \( \frac{1}{2} < z_{b,k} < \frac{3}{4} \). Finally, \( z_{b,2b} = \frac{1}{2} \).
They also conjectured that $z_{b,k}$ first increases and then decreases as $k$ increases from $b$ to $2b$. Moreover, it easily follows from the first theorem that $z_{b,k}$ decreases (as the function of $b$) for $k \geq k_0$ for some $k_0$. We get a lower bound on the minimal such $k_0$.

Also, we study a behavior of distributions of sums of dependent Bernoulli random variables “near” its medians. Let $p = cn^{-3/2}$, $X_n$ — the number of P3S in $G(n,p)$. From the theorem about Poisson approximations for the numbers of copies of fixed graphs [1], $X_n$ converges to a Poisson random variable with the parameter $c^2/2$. Let
\[
\frac{1}{2} = P(X_n < b) + w_c P(X_n = b).
\]
We prove that, for every positive $c$ and every non-negative integer $b$, for $n$ large enough, the probability $P(X_n = b)$ decreases as the function of $n$. Thus, $w_c^n$ increases for $n$ large enough.

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References


Galton-Watson processes in varying environment and accessibility percolation

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This talk deals with branching processes in varying environment, namely, whose offspring distributions depend on the generations. We give sufficient conditions for survival or extinction which rely only on the first and second moments of the offspring distributions. We show that conditions for survival relying only on the first moments of the offspring distributions cannot exist. We provide some examples and counterexamples. The techniques are based on a suitable comparison between the process and a time-homogeneous branching process with infinitely many types. These results can be applied to branching processes in varying environment with selection where every particle has a real-valued label and labels can only increase along genealogical lineages; we obtain analogous conditions for survival or extinction. These last results can be interpreted in terms of accessibility percolation on Galton-Watson trees, which represents a relevant tool for modeling the evolution of biological populations.
Selfdecomposable point processes

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Classical limit schemes involves superposition of independent point processes (PPs) which are subject to independent thinning to converge. It has been known for a few decades that all possible limit laws in the triangular array scheme constitute the class of infinitely-divisible PPs, see, e.g., [1]. When the superposed PPs are i.i.d., the weak limits are the so-called thinning-stable PPs which are Cox processes driven by a stable parameter measure. Their full description has been recently obtained in [2]. If the superposed thinned PPs are independent, but not necessarily identically distributed, the limit law is that of a thinning selfdivisible (TSD) PP which satisfy the following equality in distribution: a PP is TSD if for any \( c \in (0, 1) \) there is a point process \( c \) independent of \( X \) such that \( X = c + c \), where \( c \) stands for the operation of independent thinning with retention probability \( c \).

We present a series decomposition which fully characterises TSD PPs and discuss possible extensions to a general branching operation on the PPs which include the independent thinning as a particular case [3].

References


On convergence rate of distribution of non-discrete backward renewal processes

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We consider the renewal process \( R_t \) defined as
\[
R_t = \sum_{n=0}^{\infty} \left( \sum_{j=0}^{n} \zeta_j < t \right),
\]
where \( \zeta_i \) are mutually independent random variables, and \( P\{\zeta_i \leq s\} = F(s) \) for \( i \in \mathbb{N} \) (i.e. the distribution function of \( \zeta_0 \) can be not \( F(s) \)). For \( t \in (t_i, t_{i+1}) \), where \( t_i = \sum_{j=0}^{i} \zeta_j \), we denote \( x_t = t - t_i = t - R_{t_i} \) (backward renewal time). The backward renewal process \( X_t \) is Markov.

It is well known, that the backward renewal processes and a knowledge about its convergence rate to the stationary distribution are very important in the queueing theory. Asymptotic estimates of the convergence rate of the distribution of the backward renewal process are well known. Namely, if the condition (i) \( \{E \zeta^k < \infty, \ k > 1\} \) or (ii) \( \{E \exp(a \zeta) < \infty, \ a > 0\} \) is satisfied, then (correspondingly) for all \( \kappa < k - 1 \) or \( \alpha < a \) the following inequality is true for \( A \in \sigma(X) \) and all \( t > 0 \): (i') \( |P\{X_t \in A\} - P(A)| < K(\kappa) t^{-\kappa} \) or (ii') \( |P\{X_t \in A\} - P(A)| < K(\alpha) e^{-\alpha t} \), where \( P \) is the stationary distribution of \( X_t \), and \( K(\cdot) \) are some (unknown) constants. Our goal is to find strong bounds for these constants.
Unlike the usual situation, we suppose that the process $X_t$ starts from arbitrary initial state $X_0 = a \in \mathbb{R}_+$, and

$P\{\zeta_0 \leq s \mid X_0 = a\} = F_a(s) \overset{\text{def}}{=} 1 - \frac{1 - F(a+s)}{1 - F(a)}$; denote $P\{\zeta^a \leq s\} = F_a(s)$.

If $E\zeta_t < \infty$ and the distribution $F(s)$ is not lattice, then for all initial states $X_0$ the distribution of $X_t$ weakly converges to the stationary distribution $P_t$, i.e. $\lim_{t \to \infty} P_t \{X_t \in A\} = \lim_{t \to \infty} P_t^A = P(A)$ for all $A \in \mathcal{B}(\mathbb{R}_+)$.

**Remark.**

It is easy to give an upper bounds for $f(s) = F'(s) > 0$.

**Notation.**

\[
\Theta \overset{\text{def}}{=} \frac{E\zeta^2}{E\zeta}, \quad \tilde{f}(s) \overset{\text{def}}{=} (1 - F(s))(E\zeta)^{-1}, \quad \varkappa_{a,s} \overset{\text{def}}{=} \int_{(a,s]} \min(F'(x), F'_a(x)) \, dx;
\]

\[
\gamma_a \overset{\text{def}}{=} \inf_{a \in [0,R]} \varkappa_{a,s}, \quad \tilde{\varkappa}_a \overset{\text{def}}{=} \int \min(F'(x), \tilde{f}(x)) \, dx; \quad \tilde{\Phi}_a(s) \overset{\text{def}}{=} \varkappa_{a,s}^{-1} \tilde{\varkappa}_a; \quad P\{\tilde{\varkappa}_a \leq s\} = \tilde{\Phi}_a(s).
\]

Let $\varphi(s)$ be nondecreasing positive function.

**Theorem.** Let $E\zeta^2 < \infty$, and for almost all $t > 0$, $f(s) = F'(s) > 0$. Then for all $R > \Theta$, $\|P_t^R - P_t^0\|_{TV} \leq \frac{\mathcal{R}(x,y,R)}{\varphi(t)}$, where $\mathcal{R}(x,y,R) = \sup_{a \in [0,R]} \sum_{n=0}^{\infty} q^n \mathbb{E} \varphi \left( \sum_{m=0}^{n} \zeta_m + \zeta^2 + \zeta^y + \tilde{\varkappa}_a \right)$, and $q = \varkappa_{R}(1 - F(R))(1 - \Theta R^{-1})$.

**Corollary.** If $K(x, R) \overset{\text{def}}{=} \int_{0}^{\infty} \mathcal{R}(x,y,R) \mathbb{P}(dy) = \int_{0}^{\infty} \mathcal{R}(x,y,R) \tilde{f}(y) \, dy < \infty$, then $|P_t^R - P(A)| \leq \frac{\mathcal{K}(x,R)}{\varphi(t)}$.

**Remark.** In condition (i), for $\varphi(t) = t^\kappa$, $K(x,R) < \infty$ for $\kappa \leq k - 1$.

In condition (ii), for $\varphi(t) = e^{at}$, $K(x,R) < \infty$ if $\mathbb{E} e^{K_1} < 1 - q$ and $\mathbb{E} e^{a\zeta} < \kappa$.  

**Theorem.** Let $E\zeta < \infty$ and for some $b > a > 0$ for all $t \in (a,b)$ there exists $f(s) = F'(s) > 0$. Then $|P_t^R - P(A)| \leq \frac{\tilde{\mathcal{R}}(x)}{\varphi(t)}$, where $\tilde{\mathcal{R}}(x) = \sum_{n=0}^{\infty} \tilde{\varkappa}^n \mathbb{E} \varphi \left( \sum_{m=0}^{n} \zeta_m + \zeta^2(1) + \tilde{\varkappa}_a \right)$.

**Remark.** In condition (i), for $\varphi(t) = t^\kappa$, $\tilde{\mathcal{R}}(x) < \infty$ for $\kappa \leq k$.

In condition (ii), for $\varphi(t) = e^{at}$, $\tilde{\mathcal{R}}(x) < \infty$ if $\mathbb{E} e^{a\zeta_1} < 1 - \tilde{\varkappa}$ and $\mathbb{E} e^{a\zeta} < \kappa$.

**Remark.** It is easy to give an upper bounds for $\tilde{\mathcal{R}}(x,y,R), K(x,R)$, and $\tilde{\mathcal{R}}(x)$.

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